Total Points = 20

Quiz 6 SOLUTIONS

1. An electric motor.

(3) (a) Armature circuit ✓ | Note to grader - students may use similar terms.

(b) Circuit for electro magnet ✓

(c) Armature and shaft ✓

2. Circuit (a):

$$e_a(t) = i_A R_A + e_m = i_A R_A + \alpha \omega \quad (1)$$

(4) Armature (c):

$$J\dot{\omega} = -B\omega - \tau_L + \tau_e = -B\omega - \tau_L + \alpha i_A \quad (2)$$

(Note: can write $e_m = \alpha\omega$ and $\tau_e = \alpha i_A$ as separate equations)

3. Laplace Transforms (1) ⇒

$$E_a(s) = I_A R_A + \alpha \Omega(s) \quad \text{or} \quad I_A = \frac{E_a - \alpha\Omega}{R_A} \quad (3)$$

(2) & (3) ⇒ $(\tau_L = 0)$

$$J s \Omega(s) + B\Omega(s) = \alpha I_A = \frac{\alpha E_a - \alpha^2 \Omega}{R_A} \quad (4)$$

(4)

$$\therefore \left(Js + B + \frac{\alpha^2}{R_A}\right)\Omega(s) = \frac{\alpha E_a(s)}{R_A}$$

or

$$\Omega(s) = \frac{\alpha E_a(s)}{R_A\left(Js + B + \frac{\alpha^2}{R_A}\right)}$$

$$T(s) = \frac{\alpha}{R_A} \frac{1}{\left(Js + B + \frac{\alpha^2}{R_A}\right)} \quad (5)$$

4. $e_a(t) = E_0 H(t) \Rightarrow E_a(s) = \frac{E_0}{s}$ ✓

$$\Omega(s) = \frac{\alpha E_0}{R_A s \left(Js + B + \frac{\alpha^2}{R_A}\right)} = \frac{A}{s} + \frac{B}{\left(s + \frac{B + \frac{\alpha^2}{R_A}}{J}\right)}$$ ✓

$$A = \lim_{s \to 0} s \cdot \Omega(s) = \frac{\alpha E_0}{R_A \left(B + \frac{\alpha^2}{R_A}\right)}$$ ✓

(7) $$B = \lim_{s \to -\frac{B + \alpha^2/R_A}{J}} \left(s + \frac{B + \frac{\alpha^2}{R_A}}{J}\right) \Omega(s) = \frac{\alpha E_0}{J R_A \left(-\frac{B + \alpha^2/R_A}{J}\right)}$$ ✓

$$\therefore \Omega(s) = \frac{\alpha E_0}{R_A \left(B + \frac{\alpha^2}{R_A}\right)} \left(\frac{1}{s} - \frac{1}{s + 1/\tau}\right)$$ ✓

where $\tau = J / \left(B + \frac{\alpha^2}{R_A}\right)$ ✓

$$\therefore \omega(t) = \frac{\alpha E_0}{R_A \left(B + \frac{\alpha^2}{R_A}\right)} \cdot \left[1 - e^{-t/\tau}\right]$$ ✓

↑ alternately, H(t) may be used instead of 1. Also, they may have left $\tau = J / \left(B + \frac{\alpha^2}{R_A}\right)$ ✓

(1) 5. $\tau = \frac{J}{B + \frac{\alpha^2}{R_A}}$ ✓

(1) 6. Acceptable answers: Tachometer or generator.

Solution for Commonly Occuring Error.

2. Equation (2) is incorrectly writen

$$J\ddot{\omega} = -B\dot{\omega} - \tau_L - \tau_e = -B\dot{\omega} - \tau_L - \alpha i_A \qquad (2e)$$

Grader : −1 for writing 2e rather than (2).

3. Equation (4) becomes

$$J s^2 \Omega + B s \Omega = \alpha I_A = \frac{\alpha E_A - \alpha^2 \Omega}{R_A} \qquad (4e)$$

$$(4) \Rightarrow \left(J s^2 + B s + \frac{\alpha^2}{R_A} \right) \Omega(s) = \frac{\alpha E_A(s)}{R_A}$$

$$\Rightarrow \frac{\Omega(s)}{E_A(s)} = \frac{\alpha}{R_A} \cdot \frac{1}{\left(J s^2 + B s + \frac{\alpha^2}{R_A} \right)} \qquad (5e)$$

4. Attempt to find inverse transform.

Find poles $\quad s = \dfrac{-B \pm \sqrt{B^2 - 4\frac{J\alpha^2}{R_A}}}{2J} = s_1, s_2$

$$E_A(s) = \frac{E_0}{s}$$

$$\Rightarrow$$

$$\Omega(s) = \frac{\alpha E_0}{J \cdot R_A\, s\, (s - s_1)(s - s_2)}$$

$$= \frac{A}{s} + \frac{B}{s - s_1} + \frac{C}{s - s_2} \qquad \text{(Partial Fractions)}$$

Graders: Please give partial credit for taking a shot at the inverse transform.

Solution for COE

$$A = \frac{\alpha E_0}{J \cdot R_A \, s_1 \cdot s_2}$$

(7)

$$B = \frac{\alpha E_0}{J R_A} \frac{1}{s_1 \cdot (s_1 - s_2)}$$

$$C = \frac{\alpha E_0}{J R_A} \frac{1}{s_2 \, (s_2 - s_1)}$$

Then $\omega(t) = \frac{\alpha E_0}{J R_A} \left(\frac{1}{s_1 s_2} + \frac{e^{s_1 t}}{s_1 (s_1 - s_2)} + \frac{e^{s_2 t}}{s_2 (s_2 - s_1)} \right.$

5. There are two time constants

(1) $\tau_1 = -\frac{1}{s_1}$, $\tau_2 = -\frac{1}{s_2}$