Total Points = 11. 11

24-352 Dynamic Systems and Control: QUIZ 4

Close book and notes. You have 45 minutes to complete the following questions.

NAME: Solutions 12 March 2001

Consider the circuit shown in Figure 1. Assume that L = 3, C = 1/6, and R = 2.

1. Find the circuit's impedance.
2. Assume that the circuit has zero initial values. What is the Laplace transform of the total current flowing through the circuit, I(s), in terms of the Laplace transform of the input voltage $E_i(s)$?
3. Suppose that the voltage source is a 3 Volt DC battery. At t = 0, an open switch in the circuit is closed so that the 3 Volts is suddenly applied to the circuit. Use your impedance and Laplace transform theory to determine the output voltage as a function of time, t.

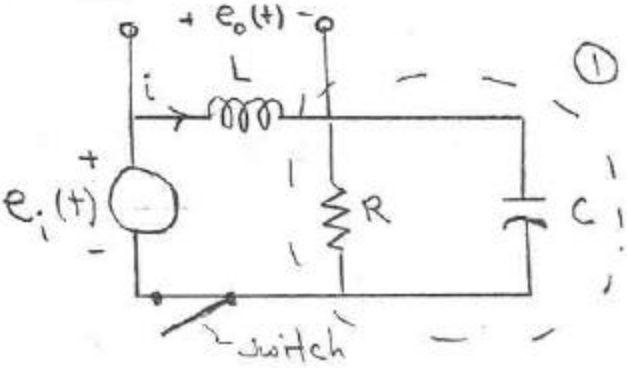

Figure 1

ANSWER

1. $Z_{1e} = \dfrac{R \cdot \frac{1}{CS}}{R + \frac{1}{CS}} = \dfrac{R}{RCS+1}$ ✓

(4) $Z_T = LS + Z_{1e} \Rightarrow$

$$Z_T = \frac{LS(RCS+1) + R}{RCS+1} = \frac{LCRS^2 + LS + R}{RCS+1}$$ ✓ ✓

f(t)	F(s)
H(t)	1/s
e^{-at}	$\frac{1}{s+a}$

Substitute values of parameters ⇒

$$Z_T = \frac{s^2 + 3s + 2}{\frac{1}{3}s + 1}$$ ✓

2. $I(s) = \dfrac{E_i(s)}{Z_T} = \dfrac{\frac{1}{3}s + 1}{s^2 + 3s + 2} \cdot E_i(s)$ ✓

(1)

3. $E_o(s) = L \cdot s \cdot I(s) = \dfrac{3s\left(\frac{1}{3}s + 1\right)}{s^2 + 3s + 2} \cdot \dfrac{3}{s} = \dfrac{3s + 9}{s^2 + 3s + 2}$ ✓

But $s^2 + 3s + 2 = (s+1)(s+2)$ ✓

$$E_o(s) = \frac{3s+9}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$ ✓ (partial fractions)

(6)

Multiply by $s+1$ & set $s = -1 \Rightarrow$

$$A = \frac{3(-1)+9}{1} = 6 \quad \checkmark$$

Multiply by $s+2$ & set $s = -2 \Rightarrow$

$$B = \frac{3(-2)+9}{(-2+1)} = -3 \quad \checkmark$$

$$\therefore \quad E_o(s) = \frac{6}{s+1} - \frac{3}{s+2}$$

From tables $\quad e_o(t) = 6e^{-t} - 3e^{-2t} \quad \checkmark$