

24-352 Dynamic Systems and Control: QUIZ 3

Close book and notes. You have 45 minutes to complete the following questions.

NAME: _____

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1. What is the governing differential equation for the system shown in Figure 1?
2. Assume: $M = 1$, $K = 2$, $B = 3$ and $f(t) = f_0 H(t)$ where $H(t)$ is the Heaviside step function. If $x(0) = 0$ and $\dot{x}(0) = 0$ find the Laplace transform of $x(t)$, $X(s)$.
3. What are the poles of $X(s)$? Plot their location in the complex s -plane.
4. Use the Laplace transform method to find $x(t)$.

ANSWER

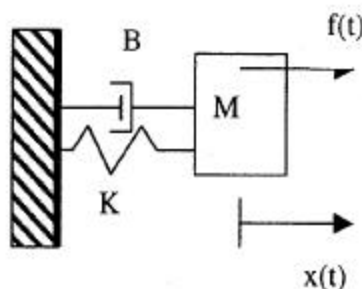


Figure 1

$$M\ddot{x} + B\dot{x} + Kx = f(t) \quad (1)$$

$$M=1, K=2, B=3, f = f_0 \cdot H(t) \Rightarrow$$

$$\ddot{x} + 3\dot{x} + 2x = f_0 H(t) \quad \checkmark$$

Laplace Transform $\Rightarrow$

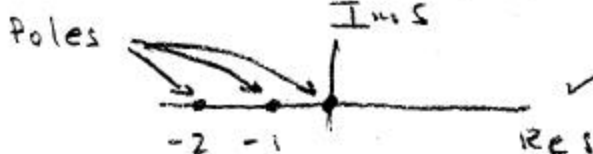
$$s^2 X(s) - s x(0) - \dot{x}(0) + 3(sX(s) - x(0))$$

$$+ 2X(s) = \frac{f_0}{s} \quad \checkmark$$

$$\Rightarrow (s^2 + 3s + 2) X(s) = \frac{f_0}{s} \Rightarrow X(s) = \frac{f_0}{s(s^2 + 3s + 2)} \quad \checkmark$$

$$3. \text{ Poles: } s(s^2 + 3s + 2) = s(s+2)(s+1) = 0 \Rightarrow s=0, s=-2, s=-1 \quad \checkmark$$

(3)



4. Use partial fractions

$$X(s) = \frac{f_0}{s(s+2)(s+1)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1} \quad \checkmark$$

$$\text{mult by } s \text{ \& let } s=0 \Rightarrow A = \frac{f_0}{2 \cdot 1} = \frac{f_0}{2} \quad \checkmark$$

$$\text{mult by } s+2 \text{ \& let } s=-2 \Rightarrow B = \frac{f_0}{(-2) \cdot (-1)} = \frac{f_0}{2} \quad \checkmark$$

$$\text{mult by } s+1 \text{ \& let } s=-1 \Rightarrow C = \frac{f_0}{(-1)(1)} = -f_0 \quad \checkmark$$

$$X(s) = f_0 \left(\frac{1}{2s} + \frac{1}{2(s+2)} - \frac{1}{s+1} \right) \Rightarrow x(t) = f_0 \left(\frac{1}{2} + \frac{1}{2} e^{-2t} - e^{-t} \right) \quad \checkmark$$