21 PTS Possible.

24-352 Dynamic Systems and Control: QUIZ 2

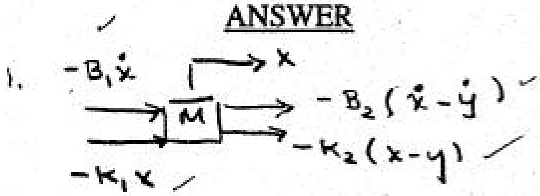
Close book and notes. You have 45 minutes to complete the following questions.

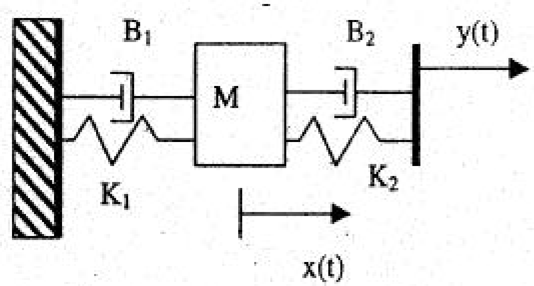
NAME: ____________________________ 12 February 2001

1. Use a free body diagram for the mass shown in Figure 1 to determine its equation of motion. (Note: the displacement y(t) is an "input" that causes the mass to move).
2. Write the equation of motion in terms of an effective stiffness K_e and damping B_e. What are they? What would the natural frequency of the undamped system be if $M = 1$, $K_1 = 1$, $K_2 = 3$, $B_1 = 0.02$, and $B_2 = 0.06$?
3. Suppose $y(t) = Y_0 \sin(\omega t)$. What do we mean by a "steady-state" solution?
4. Find the steady-state solution in terms of general values of M, B_e, K_e, K_2, B_2, Y_0, and ω.

ANSWER

1.





(5)

$$M\ddot{x} = -B_1\dot{x} - K_1 x - B_2(\dot{x}-\dot{y}) - K_2(x-y)$$

or

$$M\ddot{x} + (B_1+B_2)\dot{x} + (K_1+K_2)x = B_2\dot{y} + K_2 y$$

Figure 1

2 $M\ddot{x} + B_e\dot{x} + K_e x = B_2\dot{y} + K_2 y$, $B_e = B_1 + B_2$, $K_e = K_1 + K_2$

(3) $\omega_0 = \sqrt{\frac{K_e}{M}} = \sqrt{\frac{4}{1}} = 2$ rad/sec.

3. Several possible answers

(2) A. Harmonic response with constant amplitude & phase

B. The particular solution to the ODE.

C. The solution after the transient from the initial conditions has died off because of damping.

4. See reverse side of paper

Equation of motion: $M\ddot{x} + B_e\dot{x} + K_e x = B_2\dot{y} + k_2 y$ (1)

Assume $y = Y_0 e^{j\omega t}$, $z = A e^{j\omega t}$ where (2)

$$M\ddot{z} + B_e\dot{z} + K_e z = B_2\dot{y} + k_2 y \quad (3)$$

Then $x = \operatorname{Im}\{z\}$ (4)

(2) and (3) $\Rightarrow$

$$\{K_e - M\omega^2 + j\omega B_e\} A e^{j\omega t} = (k_2 + j\omega B_2) Y_0 e^{j\omega t}$$

(5) $\Rightarrow$

$$A = \frac{Y_0 (k_2 + j\omega B_2)}{K_e - M\omega^2 + j\omega B_e}$$

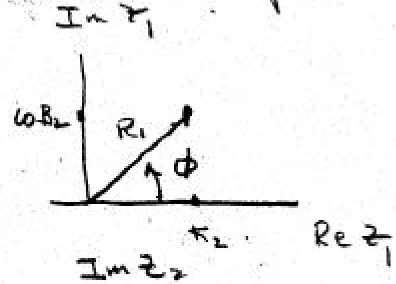
Express numerator and denominator as complex exponentials:

(4)

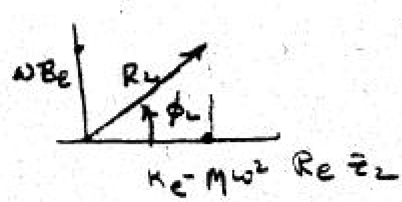
$$z_1 = k_2 + j\omega B_2 = R_1 e^{i\phi_1}$$

$$\Rightarrow R_1 = \sqrt{k_2^2 + \omega^2 B_2^2}$$

$$\phi_1 = \tan^{-1}\left(\frac{\omega B_2}{k_2}\right)$$



$$z_2 = k_e - M\omega^2 + j\omega B_e = R_2 e^{i\phi_2}$$



$$z_2 = \sqrt{(k_e - M\omega^2)^2 + \omega^2 B_e^2}$$

$$\phi_2 = \tan^{-1}\left(\frac{\omega B_e}{k_e - M\omega^2}\right)$$

(2)

Then

$$z = \frac{Y_0 R_1}{R_2} e^{j(\omega t + \phi_1 - \phi_2)}$$

and

$$x = \operatorname{Re}\{z\} = \frac{Y_0 R_1}{R_2} \sin(\omega t + \phi_1 - \phi_2)$$