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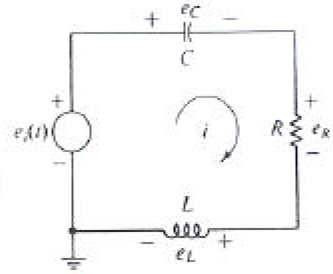
24-352 Dynamic Systems and Control: MAKE-UP QUIZ #1

Close book and notes. You have 30 minutes to complete the following questions.

NAME: 19 February 2001

The current i(t) in the electric circuit shown in the figure satisfies the following differential equation

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C} i = \dot{e}_i(t)$$



where $L = 1$, $R = 0.1$, and $C = 0.01$, and the input voltage $e_i(t) = E_0 \sin(\omega t)$. Find the steady-state current i(t) in the circuit.

ANSWER

Solution

1). 1st Approach

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$e_i = E_0 e^{j\omega t}$

Solve for z where $L\ddot{z} + R\dot{z} + \frac{1}{C} z = j\omega E_0 e^{j\omega t}$

$i = \mathrm{Im}\{z\}$

2) 2nd approach

$e_i = E_0 \sin\omega t \Rightarrow \dot{e}_i = \omega E_0 \cos\omega t$

Assume $\dot{e}_i = \omega E_0 e^{j\omega t}$

$L\ddot{z} + R\dot{z} + \frac{1}{C} z = \omega E_0 e^{j\omega t}$

Then $i = \mathrm{Re}\{z\}$

Either way gives same solution.

$z = A e^{j\omega t}$

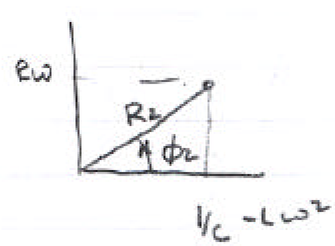
1st Approach. $\Rightarrow \left[\left(\frac{1}{C} - L\omega^2\right) + R\omega j\right] A \cancel{e^{j\omega t}} = j\omega E_0 \cancel{e^{j\omega t}}$

(2)

$$A = \frac{j\omega E_0}{\left(\frac{1}{C} - L\omega^2\right) + jR\omega}$$

Express $A = \dfrac{R_1 e^{j\phi_1}}{R_2 e^{j\phi_2}}$ $\quad R_1 = \omega E_0 \quad \phi_1 = \frac{\pi}{2}$

(5)



$R_2 = \sqrt{\left(\frac{1}{C} - L\omega^2\right)^2 + R^2\omega^2}$

$\phi_2 = \tan^{-1}\left(\dfrac{R\omega}{1/C - L\omega^2}\right)$

Then $\quad Z = \frac{R_1}{R_2} e^{j(\omega t + \phi_1 - \phi_2)}$

(2)

$$\therefore \quad i(t) = \frac{\omega E_0}{\sqrt{\left(\frac{1}{C} - L\omega^2\right)^2 + R^2\omega^2}} \sin\left(\omega t + \overset{\phi_1}{\cancel{\frac{\pi}{2}}} - \phi_2\right)$$

Since $\sin\left(x + \frac{\pi}{2}\right) = \cos x \Rightarrow$

$$i(t) = \frac{\omega E_0 \cos(\omega t - \phi_2)}{\sqrt{\left(\frac{1}{C} - L\omega^2\right)^2 + R^2\omega^2}}$$

For the values given $L = 1$, $R = 0.1$ and $C = .01$

$$i(t) = \frac{\omega E_0 \cos(\omega t - \phi_2)}{\sqrt{(100 - \omega^2)^2 + 0.01\,\omega^2}} \qquad \cos(\omega t - \phi_2)$$

$\{$ or $\sin\left(\omega t + \frac{\pi}{2} - \phi_2\right)$

where

$$\phi_2 = \tan^{-1}\left(\frac{R\omega}{1/C - L\omega^2}\right) = \tan^{-1}\left(\frac{0.1\omega}{100 - \omega^2}\right)$$