24-352 Dynamic Systems and Control: QUIZ 1

Close book and notes. You have 45 minutes to complete the following questions.

NAME: _____ 29 January 2001

Use energy principle to find the effective mass, stiffness, damping and force with respect to x(t) for the system shown in Figure 1. List your assumptions.

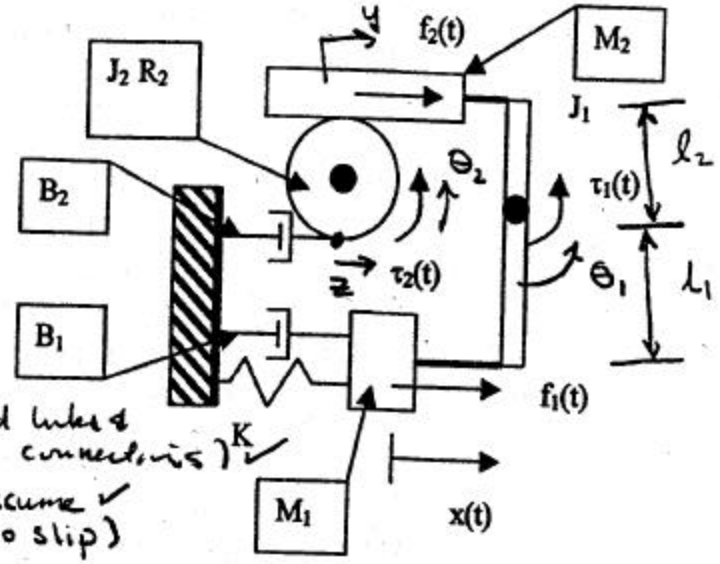

Figure 1

ANSWER

- Geometry
  Define local co-ordinates
  See figure

(7)
Assume: small rotations ✓
$x = \theta_1 l_1$, $y = -\theta_1 l_2$ (rigid links & connections) ✓
$y = -\theta_2 R_2$, $z = \theta_2 R_2$ (Assume no slip) ✓

- In terms of x:

$$\Rightarrow \frac{y}{l_2} = -\theta_1 = -\frac{x}{l_1} \Rightarrow y = -\frac{l_2 x}{l_1}$$

$$\theta_1 = \frac{x}{l_1}, \quad \theta_2 = -\frac{y}{R_2} = \frac{l_2 x}{l_1 R_2}, \quad z = -y = \frac{l_2 x}{l_1}$$

- Mass (use kinetic energy)

$$\frac{1}{2} m_e \dot{x}^2 = \frac{1}{2} M_1 \dot{x}^2 + \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} M_2 \dot{y}^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 \checkmark$$

(2)
$$m_e \dot{x}^2 = M_1 \dot{x}^2 + J_1 \left(\frac{\dot{x}}{l_1}\right)^2 + M_2 \left(\frac{l_2 \dot{x}}{l_1}\right)^2 + J_2 \left(\frac{l_2 \dot{x}}{l_1 R_2}\right)^2$$

$$\Rightarrow m_e = M_1 + \frac{J_1}{l_1^2} + M_2 \frac{l_2^2}{l_1^2} + J_2 \frac{l_2^2}{l_1^2 R_2^2} \checkmark$$

(2) • Stiffness (Potential Energy)

$$\frac{1}{2} K_e x^2 = \frac{1}{2} K x^2 \Rightarrow K_e = K \checkmark$$

(2) • Damping (Dissipation Rate): $B_e \dot{x} \cdot \dot{x} = B_1 \dot{x} \dot{x} + B_2 \dot{z} \cdot \dot{z}$

$$\Rightarrow B_e = B_1 + \left(\frac{l_2}{l_1}\right)^2 B_2 \checkmark$$

(2) • Force (work rate): $f_e \dot{x} = f_1 \dot{x} + \tau_1 \dot{\theta}_1 + f_2 \dot{y} + \tau_2 \dot{\theta}$ ✓

$$f_e = f_1 + \tau_1 / l_1 - f_2 l_2 / l_1 + \tau_2 l_2 / (l_1 R_2)$$