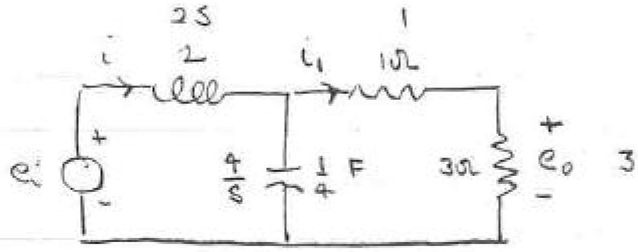
HW 8-1

5.7



From last week $Z_T = \frac{2(s^2+s+2)}{s+1}$ $\qquad E_i(s) = Z_T \cdot I(s)$

Current sharing rule.

$$I_1 = \frac{\frac{4}{s} I}{\frac{4}{s}+4} = \frac{1}{1+s} I_1(s).$$

Output voltage $E_0(s) = 3 \cdot I_1(s) = \frac{3}{1+s} \cdot \frac{E_i(s)}{Z_T}$

$$E_0(s) = \frac{3}{\cancel{1+s}} \cdot \frac{\cancel{s+1}}{2\cdot(s^2+s+2)} \cdot \frac{V_0}{s} = \frac{3}{2(s^2+s+2)} \cdot \frac{V_0}{S}$$

Poles $s=0$ $\quad s^2+s+2=0 \Rightarrow s = \frac{-1 \pm \sqrt{1-4\cdot 2}}{2}$

complex roots. $s_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$ $\quad S_1 = -\frac{1}{2}+j\frac{\sqrt{7}}{2}$ $\quad S_2 = S_1^*$

* - complex conjugate

$$E_0(s) = \frac{A}{S} + \frac{B}{S-S_1} + \frac{C}{S-S_2} = \frac{3V_0}{2} \frac{1}{S(S-S_1)(S-S_1^*)}$$

$$A = \frac{3V_0}{2} \frac{1}{S_1 S_1^*} = \frac{3V_0}{2\left(\frac{1}{4}+\frac{7}{4}\right)} = \frac{3}{4}V_0$$

$$B = \frac{3V_0}{2} \cdot \frac{1}{S_1 \cdot (S_1 - S_1^*)}$$

$$C = \frac{3V_0}{2} \frac{1}{S_1^* (S_1^* - S_1)} = B^*$$

$$\therefore\ e_o(t) = \frac{3}{4}V_o H(t) + B e^{s_1 t} + C e^{s_1^* t}$$

$$e_o(t) = \frac{3}{4}V_o H(t) + e^{-1/2 t} \cdot \left(B e^{j\frac{\sqrt{7}}{2}t} + B^* e^{-j\frac{\sqrt{7}}{2}t} \right)$$

$$= \frac{3}{4}V_o H(t) + e^{-\frac{t}{2}} \left((B + B^*) \cos\frac{\sqrt{7}}{2}t + (B - B^*) j \sin\frac{\sqrt{7}}{2}t \right.$$

$$B + B^* = 2 \cdot \mathrm{Re}\{B\}$$

$$B - B^* = 2j \cdot \mathrm{Im}\{B\}$$

Consider B: $\quad B = \frac{3V_o}{2} \frac{1}{D}$

$$D = s_1 (s_1 - s_1^*) = \left(-\frac{1}{2} + j\frac{\sqrt{7}}{2}\right) \cdot \left[\left(-\cancel{\frac{1}{2}} + j\frac{\sqrt{7}}{2}\right) - \left(-\cancel{\frac{1}{2}} - j\frac{\sqrt{7}}{2}\right)\right]$$

$$= \left(-\frac{1}{2} + j\frac{\sqrt{7}}{2}\right) j\sqrt{7}$$

$$= -\frac{\sqrt{7}}{2}j - \frac{7}{2}$$

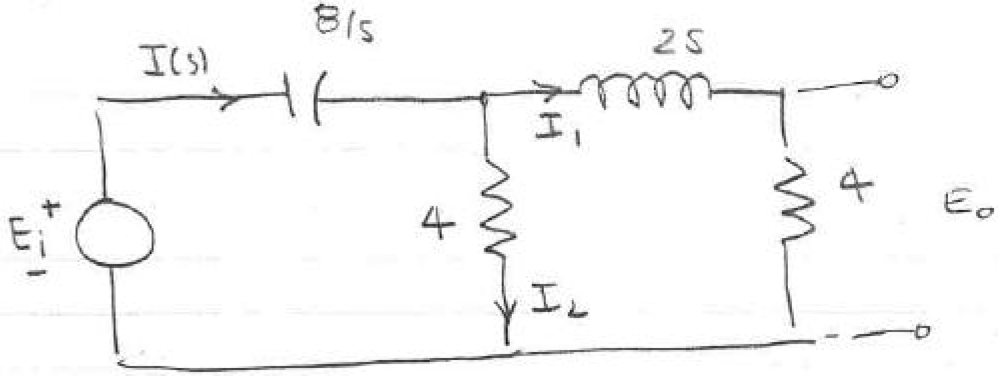
$$B = \frac{3V_o}{\cancel{2}} \frac{1}{\frac{-7 - \sqrt{7}j}{\cancel{2}}} \cdot \frac{-(7 - \sqrt{7}j)}{-(7 - \sqrt{7}j)} = \frac{3V_o(-7 + \sqrt{7}j)}{49 + 7}$$

$$B = 3V_o \frac{(-7 + \sqrt{7}j)}{56}$$

$$\therefore\ e_o(t) = \frac{3}{4}V_o \cdot H(t) + e^{-\frac{t}{2}} \left(-\frac{42}{56} V_o \cos\left(\frac{\sqrt{7}}{2}t\right) - \frac{2 \cdot 3V_o\sqrt{7}}{56} \sin\left(\frac{\sqrt{7}}{2}t\right.\right.$$

$$e_o(t) = \frac{3}{4}V_o \cdot H(t) + V_o e^{-\frac{t}{2}} \left[-\frac{3}{4}\cos\left(\frac{\sqrt{7}}{2}t\right) - \frac{3\sqrt{7}}{28} \sin\left(\frac{\sqrt{7}}{2}t\right) \right]$$

5.11.



From last week

$$Z_T = \frac{4\cdot(s^2+4s+8)}{s\cdot(s+4)}$$

$$E_i = \frac{V_0}{s} \Rightarrow \quad I(s) = \frac{V_0}{\cancel{s}} \cdot \frac{\cancel{s}\cdot(s+4)}{4\cdot(s^2+4s+8)}$$

Current Sharing:

$$I_1(s) = \frac{4\,I(s)}{2s+8} = \frac{2}{s+4}\,I(s)$$

$$E_0(s) = 4\cdot I_1(s) = \frac{8}{s+4}\cdot I(s) = \frac{8}{\cancel{s+4}}\cdot\frac{V_0\,(\cancel{s+4})}{4\cdot(s^2+4s+8)}$$

$$E_0(s) = \frac{2V_0}{s^2+4s+8}$$

Complete the square $s^2+4s+8 = (s+2)^2+4$

$$E_0(s) = V_0\left[\frac{\omega}{(s+2)^2+\omega^2}\right] \quad \text{where } \omega = 2.$$

$\Rightarrow$ (Tables).

$$e_0(t) = V_0\, e^{-2t}\sin 2t$$

Alternative Method

$$E_o(s) = \frac{A}{s} + G(s)$$

$$A = \lim_{s \to 0} sE_o(s) = \frac{3}{2 \cdot 2} = \frac{3}{4} V_o$$

$$G(s) = \frac{3V_o}{2s(s^2+s+2)} - \frac{3V_o}{4s}$$

$$= \frac{3V_o}{4} \frac{\cancel{2} - (s^2+s+\cancel{2})}{s(s^2+s+2)}$$

$$= \frac{3V_o}{4} \cdot \frac{-(s+1)}{s^2+s+2}$$

$$= -\frac{3V_o}{4} \frac{s+1}{\left(s+\frac{1}{2}\right)^2 + \frac{7}{4}}$$

$$= \frac{-3V_o}{4} \cdot \frac{Bs+C}{(s+a)+\omega^2}$$

where $B=1$, $C=1$, $a = 1/2$, $\omega = \frac{\sqrt{7}}{2}$

From tables

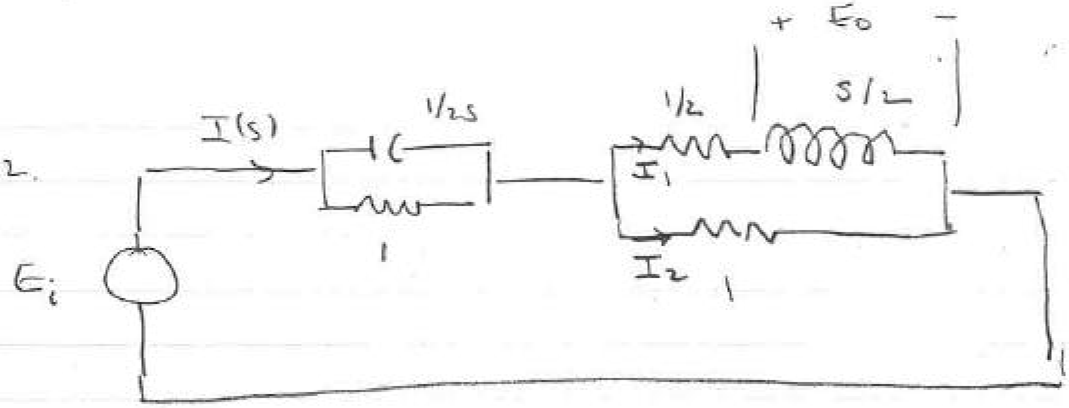
$$g(t) = e^{-t/2} \cdot \left[\cos\omega t + \frac{\left(1-\frac{1}{2}\right)}{\omega} \sin\omega t \right] \left(\frac{-3V}{4}\right.$$

$\therefore$

$$e_o(t) = \frac{3}{4} V_o H(t) - \frac{3}{4} V_o e^{-\frac{t}{2}} \left[\cos\omega t + \frac{1}{2\omega} \sin\omega t \right] \checkmark$$

$$\omega = \frac{\sqrt{7}}{2}.$$

5.12.



From last week

$$E_i(s) = \frac{2s^2+4s+4}{(1+2s)(s+3)} I(s)$$

Current sharing

$$I_1 = \frac{1 \quad I(s)}{1 + 1/2 + \frac{s}{2}} = \frac{2 \quad I(s)}{s+3}$$

$$\therefore \quad I_1(s) = \frac{\cancel{2}}{\cancel{s+3}} \cdot \frac{(1+2s)\cancel{(s+3)}}{\cancel{2}(s^2+2s+2)} E_i(s)$$

$$= \frac{1+2s}{s^2+2s+2} \frac{V_0}{s}$$

$$E_0 = \frac{s}{2} I_1(s) = \frac{V_0}{2} \cdot \frac{1+2s}{s^2+2s+2}$$

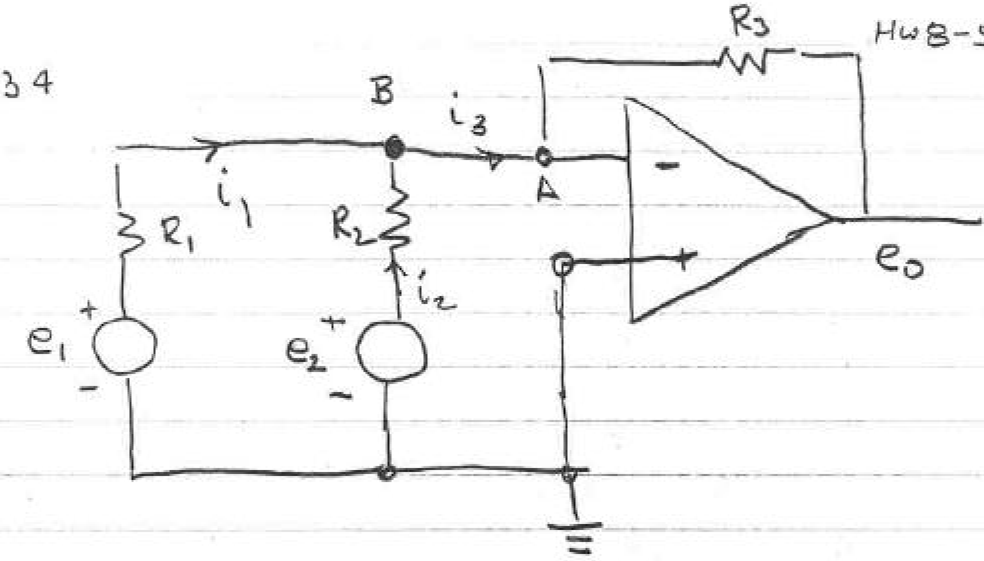
Complete the square $s^2+2s+2 = (s+1)^2+1$

$$E_0(s) = \frac{V_0}{2} \frac{2s+2-1}{(s+1)^2+1} = V_0 \frac{(s+1)}{(s+1)^2+1} - \frac{V_0}{2} \frac{1}{(s+1)^2+1}$$

$$e_0(t) = V_0 \cdot e^{-t}\left[\cos t - \frac{1}{2}\sin t\right]$$

5.34

HW8-5



Approximations

- No current flow through op-amp
- $e_A \approx 0$

Sum currents at node B.

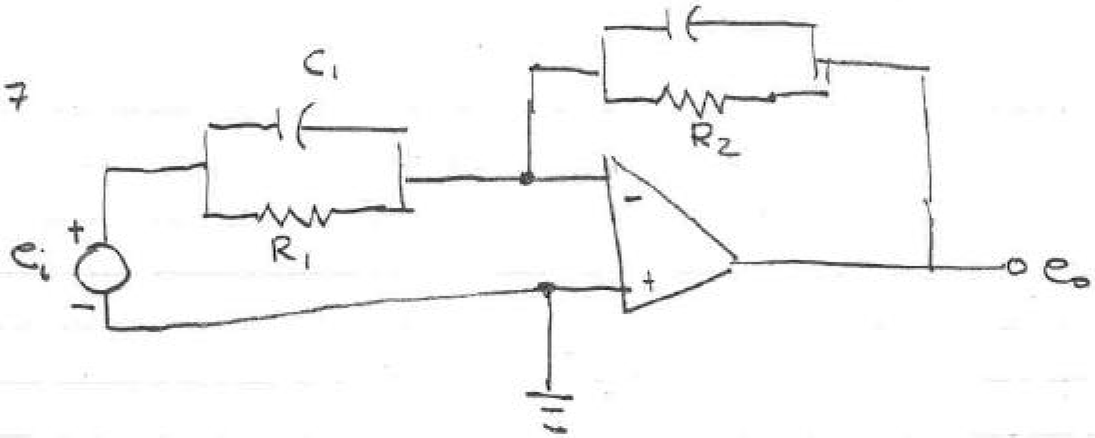
$$i_1 + i_2 = i_3$$

$$e_A \approx 0 \Rightarrow \quad i_1 = \frac{e_1}{R_1} \qquad i_2 = \frac{e_2}{R_2} \qquad i_3 = -\frac{e_0}{R_3}$$

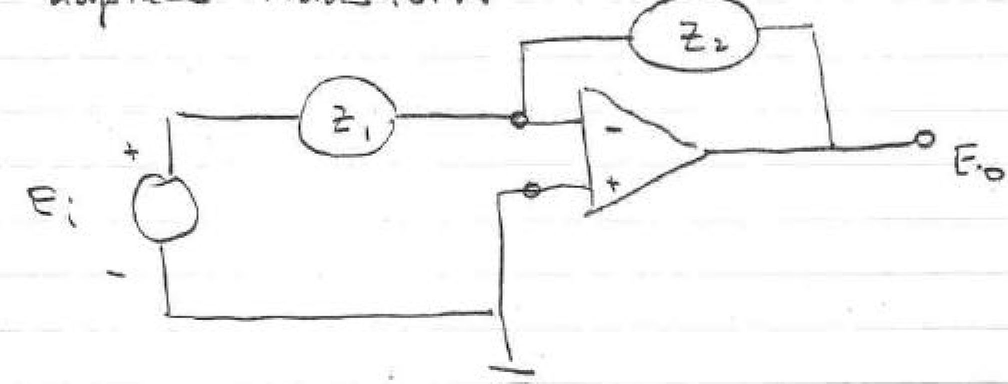
$$\therefore \quad \frac{e_1}{R_1} + \frac{e_2}{R_2} = \frac{-e_0}{R_3}$$

or
$$e_0 = -\frac{R_3}{R_1} e_1 - \frac{R_3}{R_2} e_2$$

5.37



Take Laplace Transform



Same situation as discussed in class ⇒

$$E_o(s) = -\frac{Z_2}{Z_1} E_i(s)$$

$$Z_2 = \frac{R_2 \cdot \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} = \frac{R_2}{R_2 C_2 s + 1}$$

Similarly

$$Z_1 = \frac{R_1}{R_1 C_1 s + 1} \Rightarrow E_o(s) = -\frac{R_2}{R_1} \frac{R_1 C_1 s + 1}{R_2 C_2 s + 1}$$

or $(R_1 R_2 C_2 s + R_1) E_o(s) = -(R_2 R_1 C_1 s + R_2) E_i(s)$

$$\Rightarrow R_1 R_2 C_2 \frac{de_o}{dt} + R_1 e_o(t) = -\left[R_2 R_1 C_1 \frac{de_i}{dt} + R_2 e_i(t)\right)$$