***7.10** Use (34) and the results of Example 7.1 to find the inverse transform of

$$F(s) = \frac{-s+5}{s(s+1)(s+4)}$$

Check your answer by writing a partial-fraction expansion for $F(s)$.

$$F(s) = \frac{-s+5}{s(s+1)(s+4)}$$

(34) $\quad \mathcal{L}\left\{ \int_0^t f(\lambda)\, d\lambda \right\} = \frac{1}{s} F(s)$

Ex 7.1 $\quad G(s) = \frac{-s+5}{(s+1)(s+4)} \Rightarrow g(t) = 2e^{-t} - 3e^{-4t}$

$$\therefore \quad f(t) = \int_0^t g(\lambda)\, d\lambda = \int_0^t 2e^{-\tau} - 3e^{-4\tau}\, d\tau$$

$$= \left(-2e^{-\tau} + \frac{3}{4} e^{-4\tau}\right)\Big|_0^t$$

$$f(t) = 2 - \frac{3}{4} - 2e^{-t} + \frac{3}{4} e^{-4t}$$

$$= 2(1 - e^{-t}) + \frac{3}{4}(e^{-4t} - 1)$$

Check using partial fractions

$$F(s) = \frac{A_1}{s} + \frac{A_2}{s+1} + \frac{A_3}{s+4}$$

$$A_1 = \frac{5}{1 \cdot 4}, \quad A_2 = \frac{6}{-1 \cdot 3} = -2, \quad A_3 = \frac{9}{-4(-3)} = \frac{3}{4}$$

$$\Rightarrow f(t) = \frac{5}{4} - 2e^{-t} + \frac{3}{4}e^{-4t}$$ ✓

*7.14 For a given system described by the equation $\ddot{y} + 3\dot{y} + 2y = u(t)$, use the Laplace transform to find $y(t)$ for $t > 0$ when the input is $u(t) = 5t$ for $t > 0$ and the initial conditions are $y(0) = 1$ and $\dot{y}(0) = -1$. Sketch $y(t)$.

Given $\ddot{y} + 3\dot{y} + 2y = u(t)$. $u(t) = 5t$, $y(0) = 1$, $\dot{y}(0) = -1$.

Take L.T

$$s^2 Y(s) - s\,y(0) - \dot{y}(0) + 3\left[sY(s) - y(0)\right] + 2Y(s) = \frac{5}{s^2}$$

$$\Rightarrow \left[s^2 + 3s + 2\right] Y(s) = \frac{5}{s^2} + s\,y(0) + \dot{y}(0) + 3y(0)$$

$$= \frac{5}{s^2} + s - 1 + 3$$

$$= \frac{5}{s^2} + s + 2$$

$$\therefore \quad Y(s) = \frac{5}{s^2}\frac{1}{(s^2+3s+2)} + \frac{s+2}{s^2+3s+2}$$

$$= Y_1(s) + Y_2(s)$$

Poles

$$s^2 + 3s + 2 = (s+2)(s+1)$$

$\Rightarrow$ Poles at $s = -2$ $s = -1$. Also Y_1 has a doubl pole at $s = 0$.

$$Y_2(s) = \frac{\cancel{s+2}}{(s+1)\cancel{(s+2)}} = \frac{1}{s+1} \Rightarrow y_2(t) = e^{-t}$$

$$Y_1(s) = \frac{5}{s} \cdot \frac{1}{s(s+2)(s+1)} = \frac{5}{s} \cdot G(s)$$

where $G(s) = \dfrac{1}{s \cdot (s+2)(s+1)} = \dfrac{A}{s} + \dfrac{B}{s+2} + \dfrac{C}{s+1}$

$$A = \frac{1}{2}, \quad B = \frac{1}{-2(-1)} = \frac{1}{2} \quad C = \frac{1}{-1 \cdot 1} = -1$$

$\therefore$

$$g(t) = \frac{1}{2} + \frac{1}{2} e^{-2t} - e^{-t}$$

$$y_1(t) = 5 \int_0^t g(\lambda)\, d\lambda$$

$$y_1 = 5 \left(\frac{\lambda}{2} - \frac{e^{-2\lambda}}{4} + e^{-\lambda} \right) \Big|_0^t$$

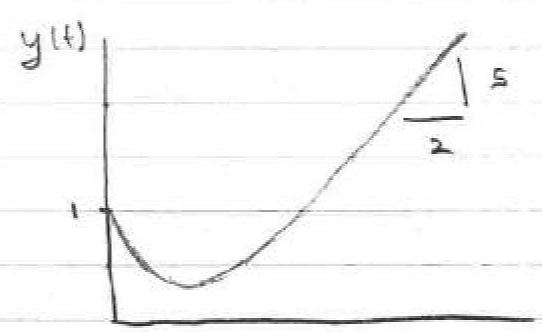
$$y_1(t) = 5 \left[\frac{t}{2} + \frac{1}{4}(1 - e^{-2t}) + e^{-t} - 1 \right]$$

Then

$$y(t) = y_1(t) + e^{-t} = 5\frac{t}{2} + \frac{5}{4}(1 - e^{-2t}) + 6e^{-t} - 5$$

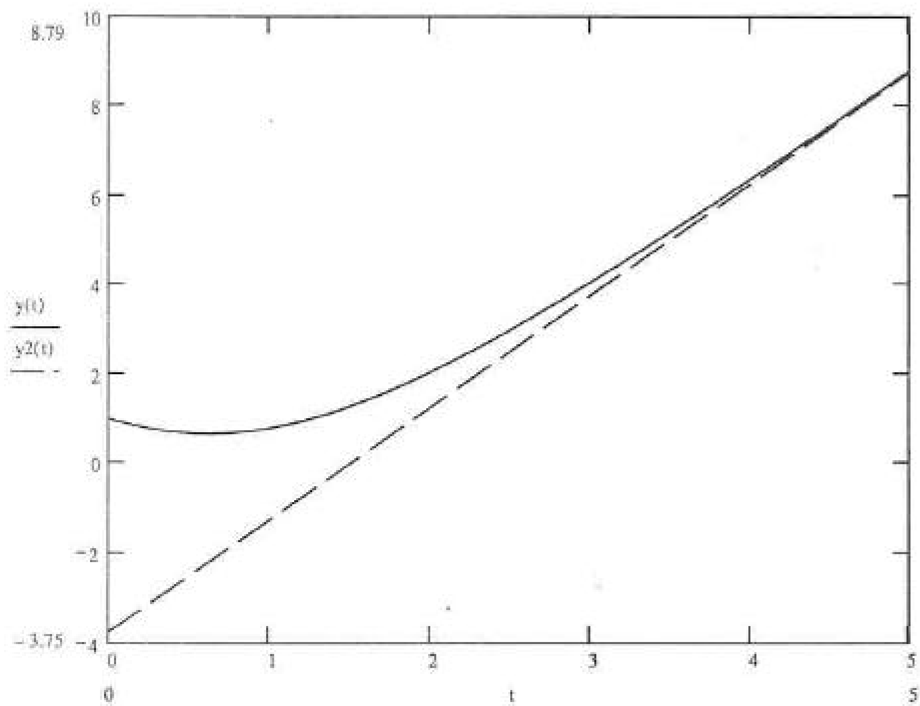
Check: $y(0) = 6 - 5 = 1$ ✓

$$y'(0) = \frac{5}{2} + \frac{5}{4} \cdot 2 - 6 = -1 \checkmark$$



$$y(t) := \left[\left(6\cdot e^{-1} - 5\right) + \frac{5\cdot\left(1 - e^{-2\cdot t}\right)}{4} + t\cdot\frac{5}{2}\right]$$

$$y2(t) := \left(-5 + \frac{5}{4} + 5\cdot\frac{t}{2}\right)$$



$$s := \frac{(y(.001) - y(0))}{.001}$$

$$s = -0.999$$

7.34 Sketch the time functions corresponding to each of the following Lapl transforms.

a) $F(s) = \frac{1}{s^2+1}\left(1+\epsilon^{-\pi s}\right)$

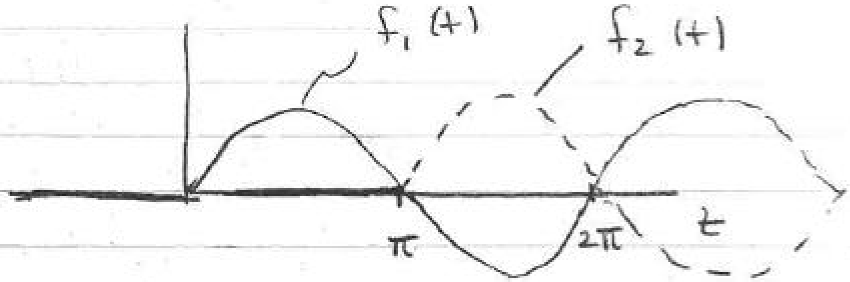
b) $F(s) = \frac{1}{s^2}\left(1-2\epsilon^{-s}+\epsilon^{-2s}\right)$

a) $F(s) = \frac{1}{s^2+1}\left(1+e^{-\pi s}\right) = F_1(s) + F_2(s)$

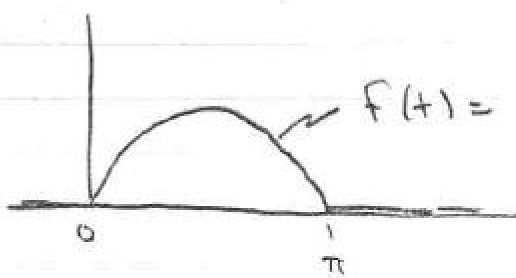
where $F_1(s) = \frac{1}{s^2+1} \Rightarrow f_1(t) = \sin t \quad t>0$

$F_2(s) = \frac{1}{s^2+1} \cdot e^{-\pi s} \Rightarrow f_2(t) = \begin{cases} \sin(t-\pi) & t>\pi \\ 0 & t<\pi \end{cases}$

But $\sin(t-\pi) = -\sin t$ so.



$\therefore$



$f(t) = \begin{cases} \sin t & 0<t<\pi \\ 0 & \text{otherwise.} \end{cases}$

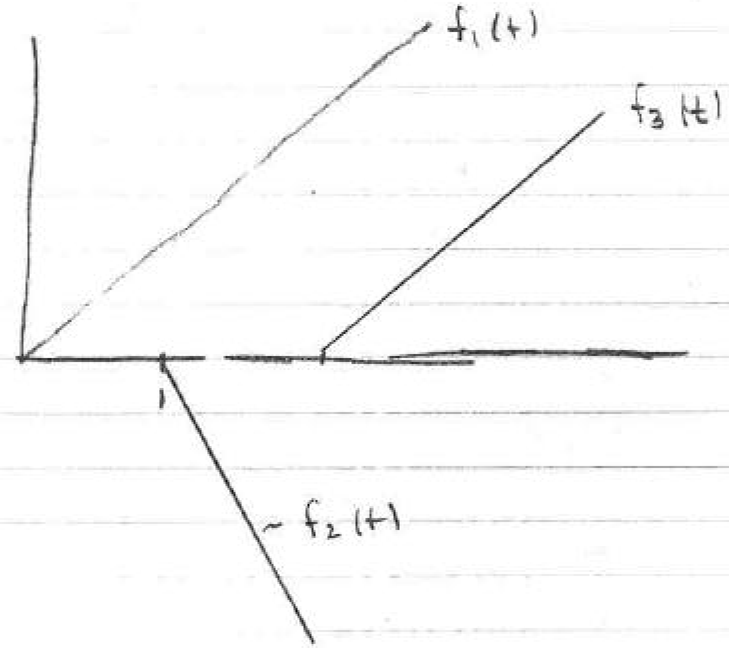
b) $F(s) = \frac{1}{s^2}\left(1 - 2e^{-s} + e^{2s}\right)$

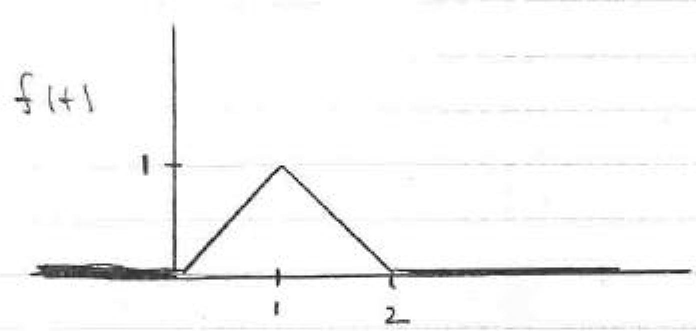
$= F_1(s) + F_2(s) + F_3(s)$

$$f_1(t) = \begin{cases} t & t > 0 \\ 0 & t < 0 \end{cases}$$

$$f_2(t) = \begin{cases} -2(t-1) & t > 1 \\ 0 & t < 1 \end{cases}$$

$$f_3(t) = \begin{cases} t-2 & t > 2 \\ 0 & t < 2 \end{cases}$$





* 7.35 Apply the initial-value and final-value theorems to find $f(0+)$ and $f(\infty)$ for each of the four transforms in Problem 7.7. If either theorem is not applicable to a particular transform, explain why this is so.

Refer to 7.7

a) $F(s) = \dfrac{s^3 + 2s + 4}{s \cdot (s+1)^2 \cdot (s+2)}$

Initial Value $\quad f(0+) = \lim_{s \to \infty} sF(s) = \lim_{s \to \infty} \dfrac{s^4}{s^4} = 1$

Final Value: All poles in LHP except for single pole at $s = 0$ ✓

$$f(\infty) = \lim_{s \to 0} sF(s) = \frac{4}{1 \cdot 2} = 2$$

b). $F(s) = \dfrac{4s^2 + 10s + 10}{s^3 + 2s^2 + 5s}$

$$f(0+) = \lim_{s \to \infty} \frac{4\cancel{s^3}}{\cancel{s^3}} = 4$$

Final value: check poles.

$D = s^3 + 2s^2 + 5s = s(s^2 + 2s + 5)$

Poles $\quad s = \dfrac{-2 \pm \sqrt{4 - 4 \cdot 5}}{2}$ $\quad$ Real part < 0 ✓

All poles in LHP except single pole at $s = 0$ ✓

$$f(\infty) = \lim_{s \to 0} sF(s) = \lim_{s \to 0} \frac{\cancel{s} \cdot 10}{5\cancel{s}} = 2$$

c. $F(s) = \dfrac{3(s^3+2s^2+4s+1)}{s(s+3)^2}$

$$f(0+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{3s^4}{s^3} \to \infty$$

solution unbounded as $t \to 0$.

Poles: single pole at $s=0$, double pole at $s=-3$ ✓

$$f(\infty) = \lim_{s\to 0} sF(s) = \frac{3\cancel{s}}{\cancel{s}\cdot 9} = \frac{1}{3}$$

d. $F(s) = \dfrac{s^3-4s}{(s+1)(s^2+4s+4)} = \dfrac{s(s^2-4)}{(s+1)(s+2)^2}$

$$F(s) = \frac{s\cancel{(s+2)}(s-2)}{(s+1)(s+2)^{\cancel{2}}} = \frac{s(s-2)}{(s+1)(s+2)}$$

$$f(0+) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{s^3}{s^2} \to \infty$$

Poles at $s=-1$, $s=-2$ ✓

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0} \frac{s^2(-2)}{2} = 0.$$

1. Take Laplace Transform. Assume $y(0) = y'(0) = 0$ (initial conditions do not affect pole locations).

$$\Rightarrow \quad (ms^2 + Bs + k)\,\bar{Y}(s) = f_0$$

or

$$\bar{Y}(s) = \frac{f_0}{m\left(s^2 + \frac{B}{m}s + \frac{k}{m}\right)}$$

Poles at

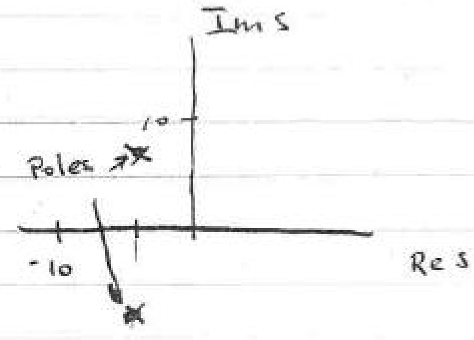
$$s_j = -\frac{B}{2m} \pm \frac{\sqrt{\frac{B^2}{m^2} - \frac{4k}{m}}}{2}$$

If $m = 1$ & $k = 100$

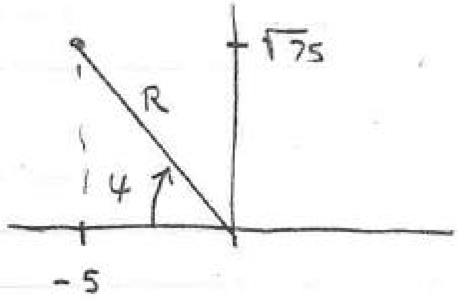
$$s_j = -\frac{B}{2} \pm \sqrt{\frac{B^2}{4} - 100} = -\frac{B}{2} \pm j\sqrt{100 - \frac{B^2}{4}}$$

If $B = 10$ $\quad s_j = -5 \pm j\sqrt{75}$

a.



b.



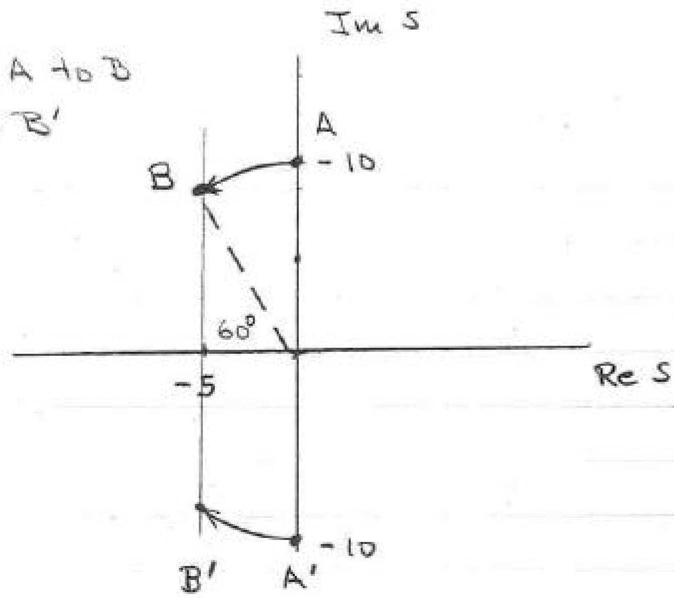
$$R = \sqrt{75 + 25} = 10$$

$$\psi = \cos^{-1}\left(\frac{1}{2}\right) = 1.047$$

From class $\omega_0 = R = 10$ rad/sec.

$$\zeta = \frac{B}{B_c} = \cos\psi = \frac{1}{2}$$

Poles move from A to B
and from A' to B'



$d\,B = 30 \Rightarrow \quad s_j = -\frac{30}{2} \pm \sqrt{\frac{900}{4} - 100}$

$= -15 \pm \sqrt{\frac{500}{4}}$

$s_j = -15 \pm 11.2 = -26.2, \; -3.8$