1. Estimate from the plots shown in Figure 1: the log decrements δ, the damping ratios ζ, the damped frequencies ω_d, and the natural frequencies ω_0 for the systems that have the two responses shown below (refer to the two systems as system Y and system Z).

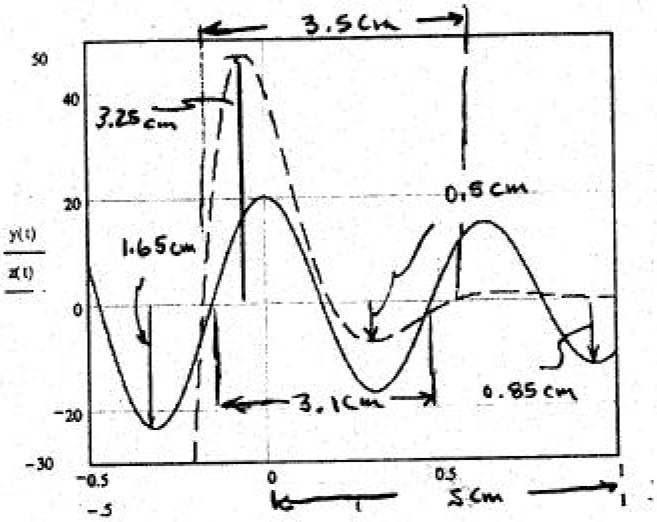


Figure 1

SYSTEM Y

1. use a ruler to measure the various dimensions of the plot in cm. y(t) is lightly damped - I use the amplitude ratio after two periods to estimate δ_Y

$$\delta_Y = \frac{1}{2} \ln\left(\frac{1.65}{0.85}\right) = \frac{1}{2} \ln(1.94) = 0.33$$

Small damping $\Rightarrow$ $\zeta_Y \simeq \frac{\delta_Y}{2\pi} = 0.052$

Damped period

$$\tau_{d_Y} = \frac{3.1\text{ cm}}{5\text{ cm/sec}} = 0.62\text{ sec}$$

$$\omega_{d_Y} = \frac{2\pi}{\tau_{d_Y}} = 10.1\text{ rad/sec}$$

$$\omega_{0_Y} = \frac{\omega_{d_Y}}{\sqrt{1-\zeta_Y^2}} = \frac{10.1}{\sqrt{1-.052^2}} = 10.1$$

negligible

SYSTEM Z: In this case use 1/2 period (Note: a full period is too inaccurate)

$$\delta_z = 2 \cdot \ln\left(\frac{3.25}{.5}\right) = 3.74$$

$$\zeta_z = \frac{\delta_z / 2\pi}{\sqrt{1 + (\delta_z / 2\pi)^2}} = 0.51 \quad \text{(formula from class)}$$

Damped Period $\tau_{dz} \simeq \dfrac{3.5\ \text{cm}}{5\ \text{cm/sec}} = 0.7\ \text{sec.}$

$$\omega_{dz} = \frac{2\pi}{\tau_{dz}} = \frac{2\pi}{0.7} = 8.98\ \text{rad/sec.}$$

$$\omega_{oz} = \frac{\omega_{dz}}{\sqrt{1-\zeta_z^2}} = \frac{8.98}{\sqrt{1 - .51^2}} = 10.4\ \frac{\text{rad}}{\text{sec.}}$$

2. Estimate the damping ratio ζ for the system that has the frequency response shown in Figure 2. Estimate the amplitude of the response if the same excitation was applied at a very low excitation frequency (close to zero).

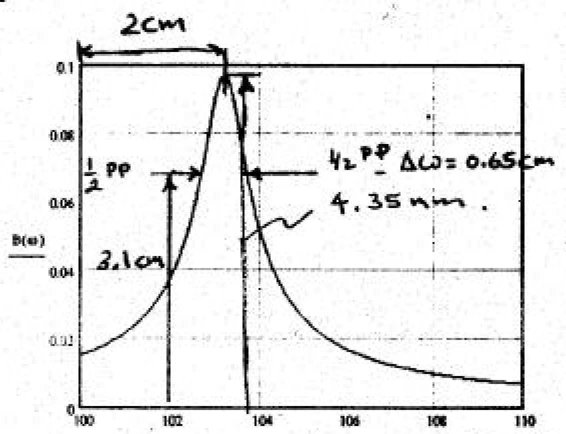


Figure 2 1.25 cm

scale 2 freq units = 1.25 cm.

$$\left.\begin{aligned} \omega_o &= 100 + \frac{2\ \text{cm}}{1.25\ \text{cm}} \cdot 2 = 103.2 \\ \Delta\omega &= 0.65\ \text{cm} \cdot \frac{2\ \text{cm}}{1.25\ \text{cm}} = 1.04 \end{aligned}\right\} \Rightarrow \zeta = \frac{1.04}{(103.2) \cdot 2} \simeq 0.005$$

For low frequency, the system responds quasi-statically and equals the peak value divided by the dynamic amplification factor, $\frac{1}{2\zeta}$. $\therefore \ \text{Amp}|_{\omega \approx 0} = 0.098 \cdot 2\zeta$

$\simeq$ ~~0.002~~ 0.001

3. Determine the steady-state response of the system shown in Figure 3 for $y(t) = Y_0 \sin(\omega t)$

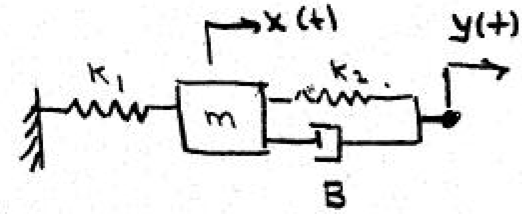


Figure 3

The equation of motion is

$$m\ddot{x} + (k_1+k_2)x + B\dot{x} = k_2 y + B\dot{y}$$

Let $k_e = k_1 + k_2$

Assume $y(t) = Y_0 e^{j\omega t}$

and solve for z where $z = Ae^{j\omega t}$ and

$$m\ddot{z} + k_e z + B\dot{z} = k_2 y + B\dot{y}$$

Then $x = \mathrm{Im}\{z\}$.

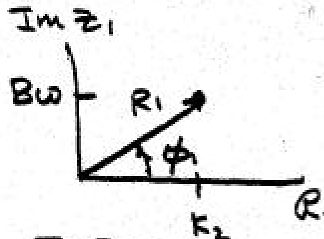
Substitute

$$(k_e - m\omega^2 + Bj\omega)A\cancel{e^{j\omega t}} = (k_2 + Bj\omega)Y_0 e^{j\omega t}$$

$$\Rightarrow \quad A = \frac{k_2 + Bj\omega}{k_e - m\omega^2 + Bj\omega} = \frac{z_1}{z_2}$$

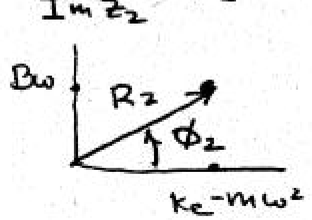
Write $z_1 = R_1 e^{j\phi_1}$

$z_2 = R_2 e^{j\phi_2}$



$$\Rightarrow \quad R_1 = \sqrt{k_2^2 + B^2\omega^2}$$

$$\phi_1 = \tan^{-1}\left(\frac{k_2}{B\omega}\right)$$



$$\Rightarrow \quad R_2 = \sqrt{(k_e - m\omega^2)^2 + B^2\omega^2}$$

$$\phi_2 = \tan^{-1}\frac{B\omega}{k_e - m\omega^2}$$

$$\therefore \quad A = \frac{R_1}{R_2}e^{j(\phi_1 - \phi_2)}, \quad z = \frac{R_1}{R_2}e^{j(\omega t + \phi_1 - \phi_2)}, \quad x = \frac{R_1}{R_2}\sin(\omega t + \phi_1 - \phi_2)$$