1. A turbine blade has a natural frequency of 500 Hz. The turbine has a speed range of 5000 to 12000 rpm. Draw a Campbell plot for this blade and circle all critical speeds. How would you change the frequency of this blade if you could alter it by no more than 10%.

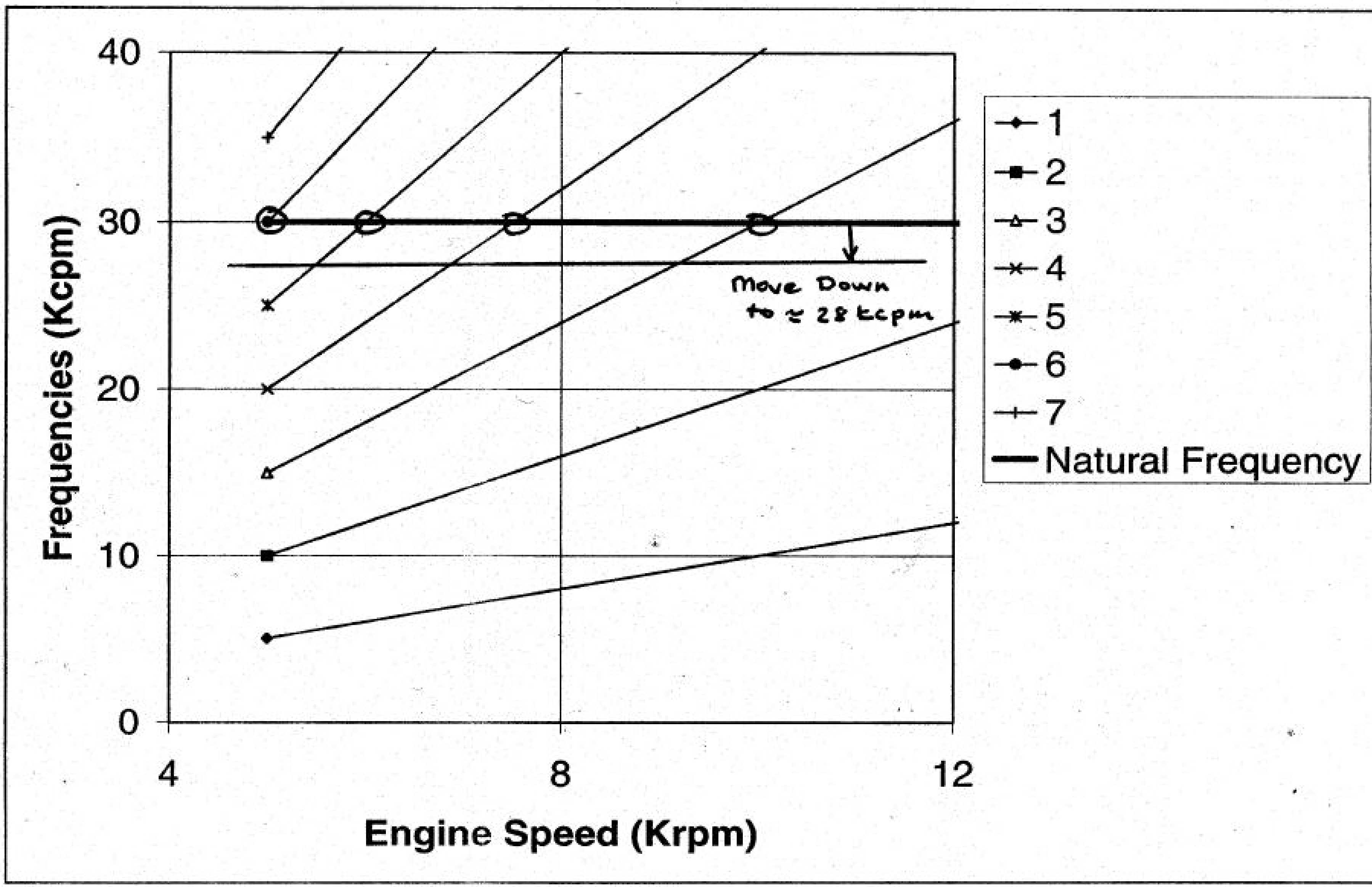
See plot on next page.

Natural frequency in cycles / minute = 500 × 60 = 30,000 cpm

Note for plot used EXCEL

Speed	1	2	3	4	5	6	7	Natural Frequency
5	5	10	15	20	25	30	35	30
15	15	30	45	60	75	90	105	30

I would change the frequency of the blade by moving it down to 28 or 27 Kcpm (1 Kcpm = 1000 cpm).


Frequencies (Kcpm)
40
30
20
10
0
4
8
12
Engine Speed (Krpm)
1
2
3
4
5
6
7
Natural Frequency
Move Down
to ≈ 28 kcpm

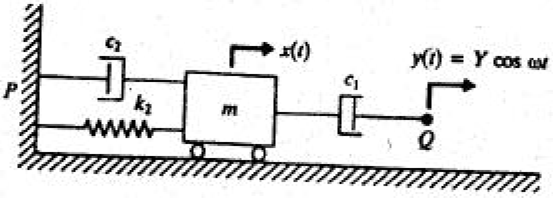
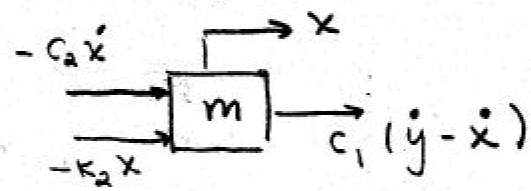


Figure 1

2. Consider the system of Figure 1. Use the complex exponential approach to determine the steady-state response to the cosine wave input.

Equation of motion

FBD



$$m\ddot{x} = -k_2 x - c_2\dot{x} + c_1(\dot{y}-\dot{x})$$

$$\Rightarrow \quad m\ddot{x} + c_e\dot{x} + k_2 x = c_1\dot{y} \qquad \text{where } c_e = c_1 + c_2 \qquad (1)$$

Excitation $\quad y = Y\cos\omega t \Rightarrow \dot{y} = -Y\omega\sin\omega t$

$\therefore \quad c_1\dot{y} = f(t) = -c_1 Y\omega\sin\omega t = f_0\sin\omega t, \quad f_0 = -c_1 Y\omega$

Note $\quad f_0\sin\omega t = \text{Im}\{f_0 e^{j\omega t}\}$

If $z(t)$ is the solution to

$$m\ddot{z} + c_e\dot{z} + k_2 z = f_0 e^{j\omega t}$$

then

$$x = \text{Im}\{z\}$$

SOLUTION

Assume $\quad z = Ae^{j\omega t}, \quad \dot{z} = j\omega Ae^{j\omega t}, \quad \ddot{z} = -\omega^2 Ae^{j\omega t}$

$$\Rightarrow \quad A\cancel{e^{j\omega t}}\left[-m\omega^2 + j\omega c_e + k_2\right] = f_0\cancel{e^{j\omega t}}$$

$$A = \frac{f_0}{k_2 - m\omega^2 + j\omega c_e}$$

We can write A in polar form as follows:

$$k_2 - m\omega^2 + j\omega c_e = R e^{j\phi}$$

where

$$R = \sqrt{(k_2 - m\omega^2)^2 + \omega^2 c_e^2}$$

$$\phi = \tan^{-1}\left(\frac{\omega c_e}{k_2 - m\omega^2}\right) \qquad 0 \le \phi \le \pi$$

Then $$A = \frac{f_0}{R} e^{-j\phi}$$

and $$z(t) = \frac{f_0}{R} e^{j(\omega t - \phi)}$$

$$x = \mathrm{Im}\{z\} = \frac{f_0}{R} \sin(\omega t - \phi)$$

Using the original notation

$$x(t) = \frac{-c_1 Y \omega \sin(\omega t - \phi)}{\sqrt{(k_2 - m\omega^2) + \omega^2 (c_1 + c_2)^2}}$$

$$\phi = \tan^{-1}\left(\frac{\omega (c_1 + c_2)}{k_2 - m\omega^2}\right) \qquad 0 \le \phi \le \pi$$

3. You may solve the problem as stated in the homework - Answer A or as re-stated in the email.- Answer B. Either solution is acceptable.

3. Suppose that in the system shown in Figure 1, $y(t) = Y_0 H(t)$ where $H(t)$ is the "Heaviside step function" defined as

$$H(t) = \begin{cases} 0 \text{ if } t<0 \\ 1 \text{ if } t \geq 0 \end{cases}$$ and that $x(0) = 0$ and $\dot{x}(0) = 0$.

Assume that the system is underdamped.

a. Find x(t).

b. If $m = k_2 = 1$, $c_1 = c_2 = 0.1$, and $Y_0=1$. Plot x(t) for $0<t<0.5$.

A. Solution to problem as stated in HW.

a. $y(t) = Y_0 H(t)$

$$m\ddot{x} + c_e\dot{x} + kx = c_1\dot{y} = c_1 Y_0 \delta(t) \qquad x(0)=0, \dot{x}(0)=0$$

(Note: as stated in class $\frac{d}{dt}(H(t)) = \delta(t)$)

Can solve $\delta(t)$ input by solving instead.

$$m\ddot{x} + c_e\dot{x} + kx = 0$$

$$x(0) = 0 \qquad \dot{x}(0) = \frac{c_1 Y_0}{m}$$

From class the solution to the underdamped system is

$$x = x_h(t) = e^{-\zeta\omega_0 t}(A\cos\omega_d t + C\sin\omega_d t)$$

Apply initial conditions

$$x(0) = 0 = A$$

$$\dot{x}(0) = \frac{c_1 Y_0}{m} = C\omega_d \Rightarrow C = \frac{c_1 Y_0}{m\omega_d}$$

$$\therefore \; x(t) = e^{-\zeta \omega_0 t} \frac{C_1 Y_0}{m \omega_d} \sin \omega_d t$$

Note $\zeta = \dfrac{C_e}{2\sqrt{m K_2}}$ $\qquad \omega_d = \sqrt{\dfrac{K_2}{m}} \cdot \sqrt{1-\zeta^2}$

$$\omega_0 = \sqrt{\frac{K_2}{m}}$$

b. See mathcad plot on HW 3-7

B. I sent the following email message

Dear Student: You have an option on problem 3. You can solve the problem as stated in the homework or you can change the specification on y(t) to

y(t) = Yo t for t > 0 and y(t) = 0 for t < 0.

(i.e y(t) is a linear function of time for t greater than zero). I think that you may find this second problem easier to solve that the one I gave you on the homework sheet – but feel free to turn in the solution to either one.

B. Replace $y(t)$ with

$$y(t) = Y_0 t \quad t > 0.$$

Then $m\ddot{x} + c_e\dot{x} + k_2 x = Y_0 \cdot c_1$ for $t > 0$

Solution $x = x_p + x_h$

$$x_p = \frac{Y_0 c_1}{k_2} \quad (\text{try it})$$

$$x_h = e^{-\zeta\omega_0 t}\left[A\cos\omega_d t + C\sin\omega_d t\right]$$

Sum and apply IC's.

$$x(t) = \frac{Y_0 c_1}{k_2} + e^{-\zeta\omega_0 t}\left[A\cos\omega_d t + C\sin\omega_d t\right]$$

$$x(0) = \frac{Y_0 c_1}{k_2} + A \Rightarrow A = \frac{-Y_0 c_1}{k_2}$$

$$\dot{x}(t) = -\zeta\omega_0 A + \omega_d C \Rightarrow C = \frac{\zeta\omega_0}{\omega_d} A = -\frac{\zeta\omega_0}{\omega_d}\frac{Y_0 c_1}{k_2}$$

$$\therefore x(t) = \frac{Y_0 c_1}{k_2} \cdot \left(1 - e^{-\zeta\omega_0 t}\left(\cos\omega_d t + \frac{\zeta\omega_0}{\omega_d}\sin\omega_d t\right)\right)$$

See plot on HW3 - 8

Homework Assignment 3

3. A.

Define variables

$$m := 1 \quad k2 := 1 \; c1 := .1 \;\; c2 := .1 \;\; Yo := 1$$

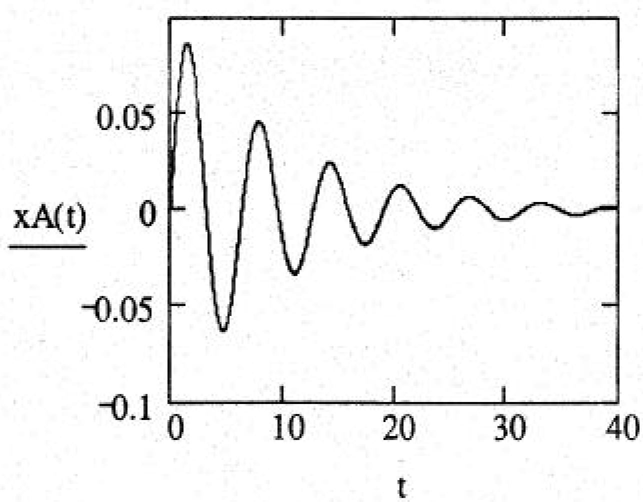
Calculate parameters:

$$Ce := c1 + c2 \qquad \zeta := \frac{Ce}{2\cdot\sqrt{k2\cdot m}}$$

$$\omega o := \sqrt{\frac{k2}{m}} \qquad \omega d := \omega o\cdot\sqrt{1-\zeta^2}$$

Solution to Statement A

$$xA(t) := c1\cdot\frac{Yo\cdot e^{-\zeta\cdot\omega o\cdot t}\cdot\sin(\omega d\cdot t)}{m\cdot\omega d}$$



Solution to Statement B

$$xB(t) := \frac{Yo}{k2}\cdot c1\cdot\left[1 - e^{-\zeta\cdot\omega o\cdot t}\cdot\left(\cos(\omega d\cdot t) + \frac{\zeta}{\omega d}\cdot\omega o\cdot\sin(\omega d\cdot t)\right)\right]$$

$$Ref(t) := Yo\cdot\frac{c1}{k2}$$