24-352 DYNAMIC SYSTEMS & CONTROL

HOMEWORK ASSIGNMENT #2

DUE 1/31/01

PROBLEMS

1. Consider the mass/spring system of Figure 1. Assume $y(t) = 0$. Let M = 1000 kg and K= 40,000 N/m. Assume f(t) is from gravity, $x(0) = 0.1$ and $\dot{x}(0) = 1\,m/s$.
   a. What is the natural frequency of the system in radians/second and in Hertz.
   b. Find x(t). Plot x(t) for $0 < t < 2$. What is the maximum value of x over this interval?

2. Consider the system of Figure 1. If $f(t) = 4000\cos(\omega t)$
   a. Find the particular solution to the governing differential equation. Determine the magnitude of the amplitude, R, and the phase of the response, ϕ, as functions of the excitation frequency ω.
   b. Plot $R(\omega)$ and $\phi(\omega)$ for $0 < \omega < 20$. Indicate on your plot of R the regions that are primarily controlled by 1) the spring and 2) the mass.

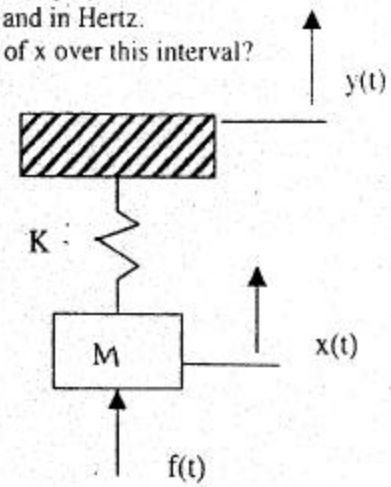

Figure 1

3. Assume that the force f(t) is the square wave shown in Figure 2

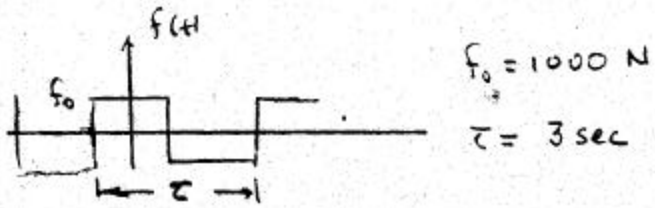

Figure 2

   a. Find the Fourier series for f(t), i.e. find F_n and G_n where

$$f(t) = \frac{F_0}{2} + \sum_{n=1}^{\infty} F_n \cos(n\omega t) + G_n \sin(n\omega t)$$

   b. Find the particular solution if the square wave is applied as a force to the system of Figure 1, i.e. assume that x(t) has the same period as f(t) and can be represented as a Fourier series of the form

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

   Find the a_n and b_n. Which frequency component of the response is the largest? Why?

4. Suppose that instead of a force being applied to the system in Figure 1 the system is excited by a sinusoidal motion at y. That is $y(t) = Y_0 e^{j\omega}$. Find the particular solution. Use your result to determine the particular solution if instead $y(t) = Y_0 \sin(\omega t)$.