1. Kinetic Energy
   a. Use the concept of equivalent kinetic energy to find the equivalent mass for the system shown in Fig. 1. That is, find M_e in terms of m_1, m_2, L1 and L2 so that the kinetic energy of the lever system is equal to $\frac{1}{2}M_e(\dot{x})^2$.
   b. Use kinetic energy to find the equivalent moment of inertia with respect to θ_1 of the system shown in Fig. 2

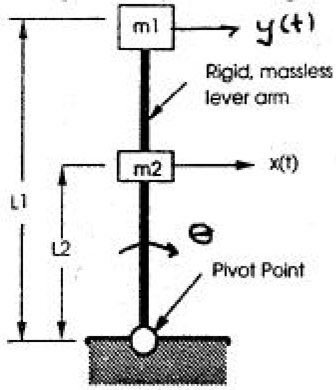


Figure 1

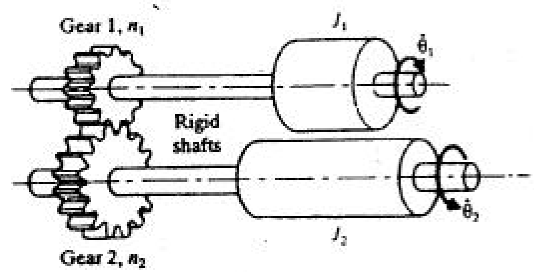


Figure 2

a. $$\frac{1}{2} m_e \dot{x}^2 = \frac{1}{2} m_1 \dot{y}^2 + \frac{1}{2} m_2 \dot{x}^2$$

Geometry: θ small $\Rightarrow$ $x = \theta \cdot L_2$, $y = \theta \cdot L_1$

$$\Rightarrow \frac{x}{L_2} = \frac{y}{L_1} = \theta$$

$$\therefore \frac{1}{2} m_e \dot{x}^2 = \frac{1}{2} m_1 \left(\frac{\dot{x} L_1}{L_2}\right)^2 + \frac{1}{2} m_2 \dot{x}^2$$

$$\Rightarrow m_e = m_2 + m_1 \left(\frac{L_1}{L_2}\right)^2$$

b. $$\frac{1}{2} J_e \dot{\theta}_1^2 = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \dot{\theta}_2^2$$

Geometry: $\dot{\theta}_1 n_1 = \dot{\theta}_2 n_2$

$$\therefore \frac{1}{2} J_e \dot{\theta}_1^2 = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \left(\frac{\dot{\theta}_1 n_1}{n_2}\right)^2$$

$$\Rightarrow J_e = J_1 + \left(\frac{n_1}{n_2}\right)^2 J_2$$

2. Potential Energy
   a. Use the concept of equivalent potential energy to find the equivalent linear stiffness with respect to x for the system shown in Fig. 3. That is, find K_e so that the potential energy stored in the system is equal to $\frac{1}{2}K_e(x)^2$.
   b. Use the concept of equivalent potential energy to find the equivalent torsional stiffness with respect to θ for the systems shown in Figures 4 and 5.

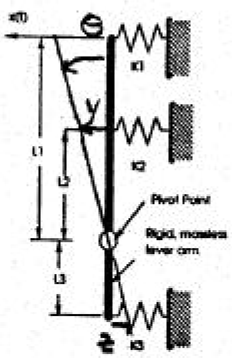


Figure 3

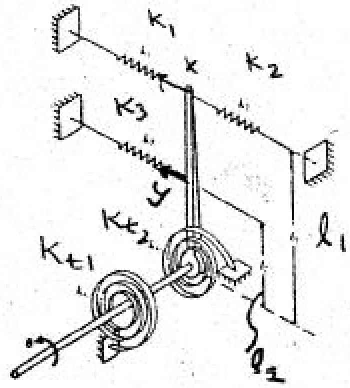


Figure 4

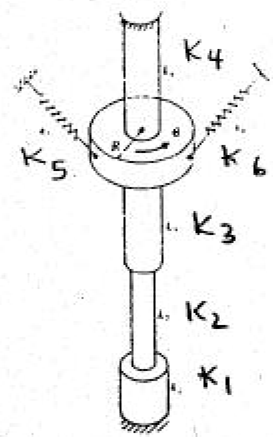


Figure 5

a. $\frac{1}{2}k_e x^2 = \frac{1}{2}K_1 x^2 + \frac{1}{2}K_2 y^2 + \frac{1}{2}K_3 z^2$

Geometry: $\Theta \simeq \frac{x}{L_1} \simeq \frac{y}{L_2} \simeq \frac{z}{L_3}$ $\quad \Theta$ small

$\Rightarrow \frac{1}{2}k_e x^2 = \frac{1}{2}K_1 x^2 + \frac{1}{2}K_2\left(\frac{L_2}{L_1}x\right)^2 + \frac{1}{2}K_3\left(\frac{L_3}{L_1}x\right)^2$

$\Rightarrow k_e = K_1 + \left(\frac{L_2}{L_1}\right)^2 K_2 + \left(\frac{L_3}{L_1}\right)K_3$

b. 1) Figure 4

$\frac{1}{2}K_{te}\Theta^2 = \frac{1}{2}K_{t_1}\Theta^2 + \frac{1}{2}K_{t_2}\Theta^2 + \frac{1}{2}(K_1+K_2)x^2$
$+ \frac{1}{2}K_3 y^2$

Geometry: $\Theta \simeq \frac{x}{\ell_1} \simeq \frac{y}{\ell_2}$

$\frac{1}{2}K_{te}\Theta^2 = \frac{1}{2}K_{t_1}\Theta^2 + \frac{1}{2}K_{t_2}\Theta^2 + \frac{1}{2}(K_1+K_2)(\ell_1\Theta)^2 + \frac{1}{2}K_3(\ell_2\Theta)$

$\Rightarrow K_{te} = K_{t_1} + K_{t_2} + (K_1+K_2)\ell_1^2 + K_3\ell_2^2$

2.b. 2) Figure 5

Springs 1, 2 and 3 act in series. The equivalent spring to replace these three has a torsional stiffness k_{e1} where from class

$$\frac{1}{k_{e_1}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

Assume k_{e_1} is known. Equivalent potential energy ⇒

$$\frac{1}{2} k_e \theta^2 = \frac{1}{2} k_4 \theta^2 + \frac{1}{2} k_{e_1} \theta^2 + \frac{1}{2}(k_5 + k_6)(R\theta)^2$$

$$\Rightarrow \quad k_e = k_4 + k_{e_1} + k_5 + k_6$$

3. Consider the system shown in Figure 6. Define a single degree of freedom system in terms of the translational motion, x(t). Use energy arguments to find the equivalent mass, damping constant, stiffness and force for the equivalent single degree of freedom system.

Simplifying Assumptions:

- The motions are small.
- The cable between the mass and the pulley may be model has no mass and infinite stiffness.
- The mass and moment of inertia of rigid link 1 are zero.

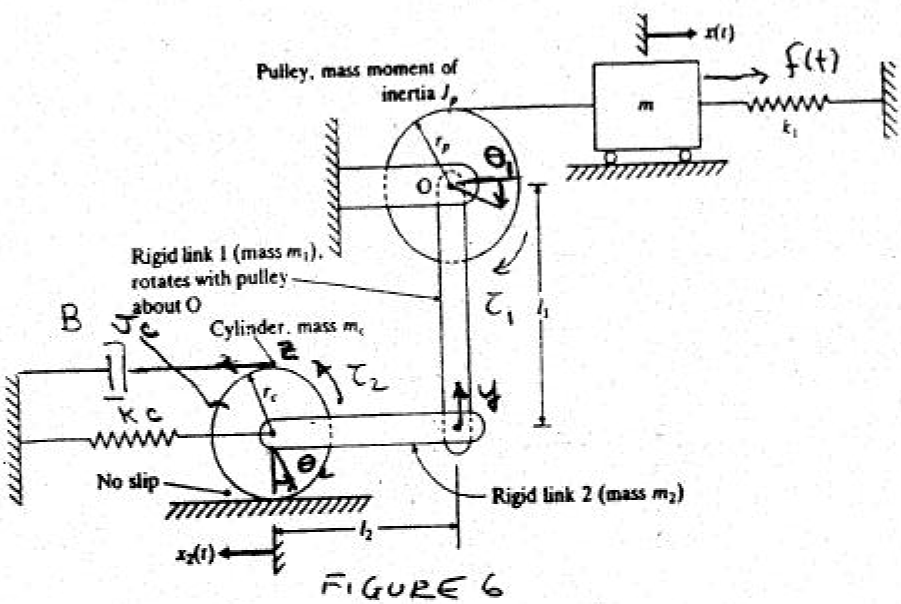
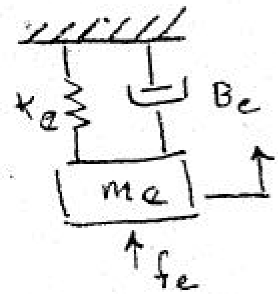


FIGURE 6

- GEOMETRY: x small



$$x \simeq r_p \Theta_1$$
$$x_2 \simeq l_1 \Theta_1 = l_1 x / r_p$$
$$\Theta_2 \simeq x_2 / r_c = l_1 x / (r_p \cdot r_c)$$
$$y \simeq 0 \Rightarrow \text{rotation of link } 2 \simeq 0$$
$$z = x_2 + r_c \cdot \Theta_2 = 2x_2 = 2 l_1 x / r_p$$

Note: The motion $y = l_1 (1 - \cos\Theta_1) \simeq \frac{l_1}{2}\Theta_1^2$

$\Rightarrow$ vertical motion y is much smaller than other motions because it is proportional to Θ_1^2.

- MASS:

$$\frac{1}{2} m_e \dot{x}^2 = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_p \dot{\Theta}_1^2 + \frac{1}{2}(m_2 + m_c)\dot{x}_2^2 + \frac{1}{2} J_c \dot{\Theta}_2^2$$

Geometry $\Rightarrow$

$$\frac{1}{2} m_e \dot{x}^2 = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_p \left(\frac{\dot{x}}{r_p}\right)^2 + \frac{1}{2}(m_2 + m_c)\left(\frac{l_1}{r_p}\dot{x}\right)^2 + \frac{1}{2} J_c \left(\frac{l_1}{r_p r_c}\dot{x}\right)^2$$

$$\boxed{m_e = m + \frac{J_p}{r_p^2} + (m_2 + m_c)\left(\frac{l_1}{r_p}\right)^2 + J_c\left(\frac{l_1}{r_p r_c}\right)^2}$$

- STIFFNESS

$$\frac{1}{2} k_e x^2 = \frac{1}{2} k_1 x^2 + \frac{1}{2} k_c x_2^2$$

Geometry $\Rightarrow$

$$\frac{1}{2} k_e x^2 = \frac{1}{2} k_1 x^2 + \frac{1}{2} k_c \left(\frac{l_1 x}{r_p}\right)^2 \Rightarrow \boxed{k_e = k_1 + k_c \frac{l_1^2}{r_p^2}}$$

- DAMPING

$$B_e \dot{x} \cdot \dot{x} = B \cdot \dot{z} \cdot \dot{z} = B(2 l_1 \dot{x} / r_p)^2 \Rightarrow \boxed{B_e = B\, 4 l_1^2 / r_p}$$

- FORCE

$$f_e \cdot \dot{x} = f \cdot \dot{x} + \tau_1 \dot{\Theta}_1 + \tau_2 \dot{\Theta}_2 = f\dot{x} + \tau_1 \frac{\dot{x}}{r_p} + \tau_2 l_1 \dot{x} / (r_p \cdot r_c$$

$$\boxed{f_e = f + \tau_1 / r_p + \tau_2 l_1 / (r_p \cdot r_c)}$$