Note to grader: don't care about $\vec{b}_2$.

- Last Week

From last week the lowest natural frequency and its mode:

$$\omega^2 = \omega_1^2 = \frac{3-\sqrt{5}}{2} = 0.382 \Rightarrow \omega_1 = 0.618 \qquad (1)$$

$$\vec{b}_1 = c_1 \begin{bmatrix} 1 \\ 1.618 \end{bmatrix}$$

- Normalize the mode so that $\vec{b}_1^T \vec{b}_1 = 1 \Rightarrow$

$$c_1^2 (1 + 1.618^2) = 1 \Rightarrow c_1 = .526$$

$$\therefore \vec{b}_1 = \begin{bmatrix} 0.526 \\ 0.85 \end{bmatrix} \quad \text{(normalized so length = 1).} \qquad (2)$$

[note: could also have $\vec{b}_1 = -\begin{bmatrix} 0.526 \\ 0.85 \end{bmatrix}$]

- Assume $\vec{x} = \Sigma$ modes & substitute

We have the equation

$$m\ddot{\vec{x}} + \mathbb{K}\vec{x} = \vec{f}(t) \qquad (3)$$

where $m = 1$, $\mathbb{K} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$, $\vec{f} = \begin{bmatrix} \sin\omega t \\ 2\sin\omega t \end{bmatrix}$

Assume $\vec{x} = a_1(t)\vec{b}_1 + a_2(t)\vec{b}_2 \qquad (4)$

$$\Rightarrow \ddot{\vec{x}} = \ddot{a}_1(t)\vec{b}_1 + \ddot{a}_2(t)\vec{b}_2 \qquad (5)$$

(3) becomes $(m=1)$

$$\ddot{a}_1(t)\vec{b}_1 + \ddot{a}_2(t)\vec{b}_2 + a_1(t)\mathbb{K}\vec{b}_1 + a_2(t)\mathbb{K}\vec{b}_2 = \vec{f} \quad (6$$

• Use Modal Equation to Simplify

But the modes satisfy the equation

$$\mathbb{K}\vec{b}_j = \cancel{m}\,\omega_j^2\,\vec{b}_j \quad \text{(7)}$$

$$(6) \Rightarrow \left[\ddot{a}_1(t) + \omega_1^2 a_1(t)\right]\vec{b}_1 + \left[\ddot{a}_2(t) + \omega_2^2 a_2(t)\right]\vec{b}_2 = \vec{f}$$

• Use Orthogonality

Multiply by $\vec{b}_1^T$, use $\vec{b}_1^T \cdot \vec{b}_2 = 0$ and $\vec{b}_1^T \vec{b}_1 = 1$

$$\left[\ddot{a}_1(t) + \omega_1^2 a_1(t)\right]\cancel{\vec{b}_1^T\vec{b}_1} = \vec{b}_1\vec{f} = [0.526 \quad 0.85]\begin{bmatrix} \sin\omega \\ 2\sin\omega \end{bmatrix}$$

$$\Rightarrow \quad \ddot{a}_1 + \omega_1^2 a_1 = (0.526 + 0.85\cdot 2)\sin\omega t = 2.2\sin\omega t \quad (8$$

• Initial Conditions

$$\vec{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = a_1(0)\vec{b}_1 + a_2(0)\vec{b}_2$$

Use orthogonality - multiply by $\vec{b}_1^T \Rightarrow$

$$[0.526 \quad 0.85]\begin{bmatrix} 1 \\ 0 \end{bmatrix} = a_1(0)(1) + \cancel{a_2(0)}\cdot(0)$$

$$\Rightarrow a_1(0) = 0.526 \quad (9)$$

Similarly

$$\dot{a}_1(0) = \vec{b}_1^T\dot{x}(0) = \vec{b}_1^T\begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0 \quad (10)$$

• Solution to (8)

$$a_1(t) = a_{1h}(t) + a_{1p}(t) \quad (11)$$

$$a_{1h}(t) = A_1\cos\omega_1 t + B_1\sin\omega_1 t \quad (12)$$

$$a_{1p}(t) = \frac{2.2\sin\omega t}{\omega_1^2 - \omega^2} \quad (13)$$

- Apply initial conditions

$$a_1(0) = 0.526 = a_{1h}(0) + a_{1p}(0) = A_1 + 0 \Rightarrow A_1 = .526$$

$$\dot{a}_1(0) = 0 = B_1\omega_1 + \frac{2.2\,\omega}{\omega_1^2 - \omega^2} \Rightarrow B_1 = \frac{-2.2\,\omega/\omega_1}{\omega_1^2 - \omega^2}$$

$$\therefore \boxed{a_1(t) = 0.53 \cos\omega_1 t + \frac{2.2}{\omega_1^2 - \omega^2}\left(\sin\omega t - \frac{\omega}{\omega_1}\sin\omega_1 t\right.}$$

where $\omega_1 = 0.62$

- Note: If student uses $\vec{b}_1 = -\begin{bmatrix} 0.53 \\ 0.85 \end{bmatrix}$

then $a_1(t)$ will have the opposite sign.