* 13.3 Evaluate the transfer functions $T_1(s) = Y(s)/U(s)$ and $T_2(s) = Z(s)/U(s)$ as rational functions for the block diagram shown in Figure P13.3.

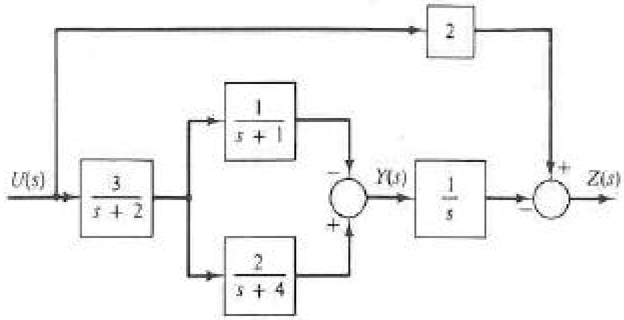


FIGURE P13.3

$T_1(s) = Y(s) / U(s)$

Blocks in parallel

$$Y(s) = U(s) \cdot \frac{3}{s+2} \left[\frac{2}{s+4} - \frac{1}{s+1} \right]$$

$$= U(s) \cdot \frac{3}{s+2} \left[\frac{2s+2 - s - 4}{(s+4)(s+1)} \right]$$

$$= U(s) \cdot \frac{3(s-2)}{(s+2)(s+4)(s+1)}$$

$$\therefore \quad T_1(s) = \frac{Y}{U} = \frac{3(s-2)}{(s+2)(s+4)(s+1)} = \frac{3s-6}{s^3 + 7s^2 + 14s + 8}$$

Note to grader: either form acceptable

$T_2 = Z(s) / U(s)$

$$Z(s) = -\frac{1}{s} Y(s) + 2U(s) = \left(-\frac{1}{s} T_1(s) + 2 \right) U(s)$$

$$\Rightarrow \quad T_2 = 2 - \frac{T_1(s)}{s} = \frac{2s^4 + 14s^3 + 28s^2 + 16s - 3s + 6}{s(s^3 + 7s^2 + 14s + 8)}$$

$$\therefore \quad T_2(s) = \frac{2s^4 + 14s^3 + 28s^2 + 13s + 6}{s^4 + 7s^3 + 14s^2 + 8}$$

or various alternatives

$$\text{e.g.} \quad T_2 = 2 - \frac{3(s-2)}{s(s+2)(s+4)(s+1)}$$

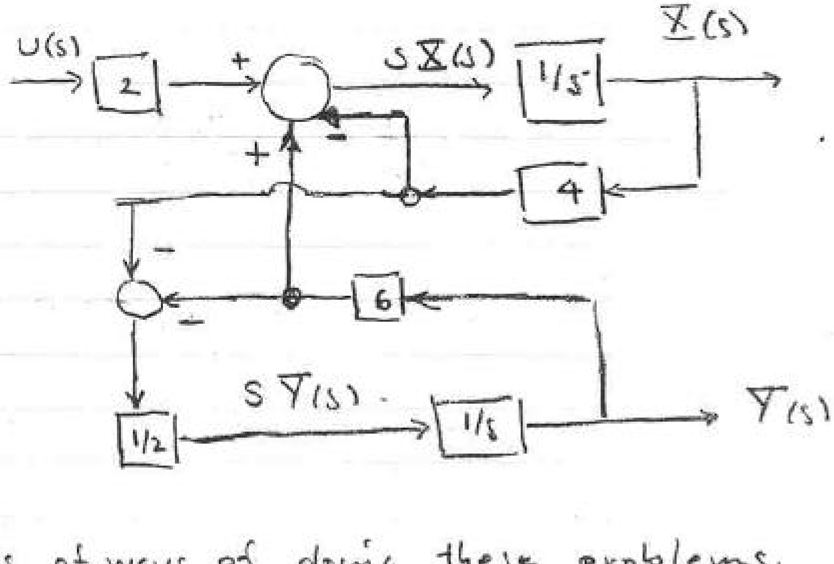
13.4 Draw block diagrams for each of the following sets of state-variable equations.

a) $\dot{x} = -4x + 6y + 2u(t)$
$\dot{y} = -2x - 3y$

b) $\dot{x}_1 = -3x_1 + 5x_2 + 3u(t)$
$\dot{x}_2 = 4x_1 - 6x_2 - u(t)$

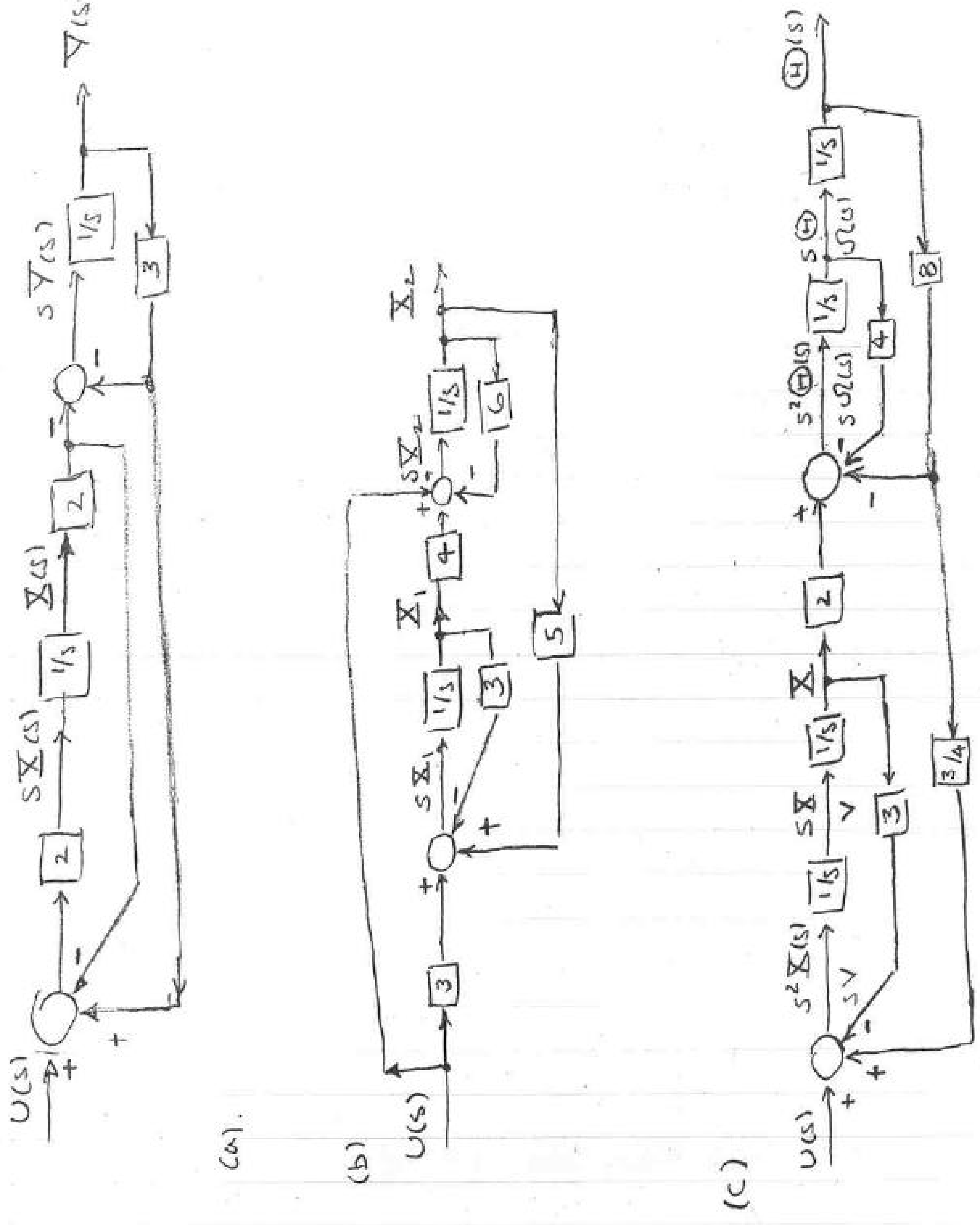
c) $\dot{\theta} = \omega$
$\dot{\omega} = -8\theta - 4\omega + 2x$
$\dot{x} = v$
$\dot{v} = 6\theta - 3x + u(t)$

a).



Lots of ways of doing these problems.

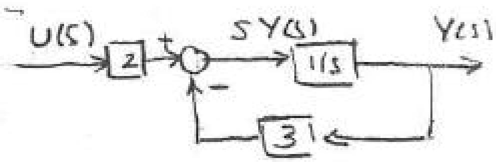
Alternative: need longer distance - use landscape.



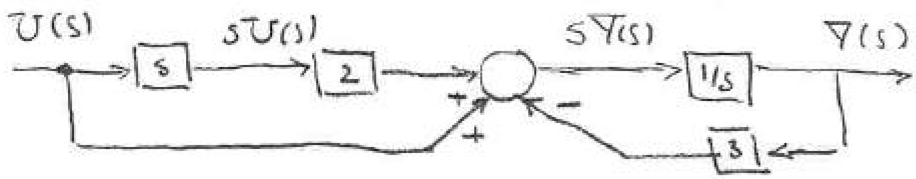
13.6 Draw block diagrams for each of the following input-output models.

a) $\dot{y} + 3y = 2u(t)$
b) $\dot{y} + 3y = 2\dot{u} + u(t)$
c) $\dddot{z} + 4\dot{z} + 2z = u(t)$
d) $\dddot{z} + 4\dot{z} + 2z = 3\dot{u} + 2u(t)$

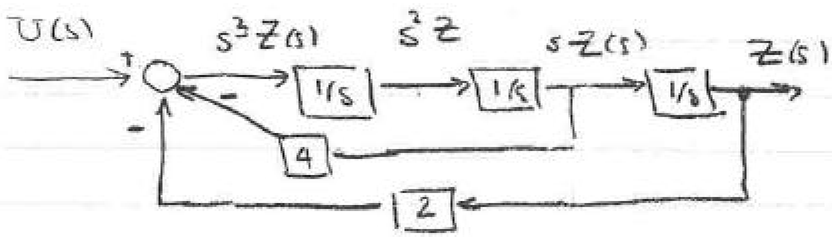
(a)



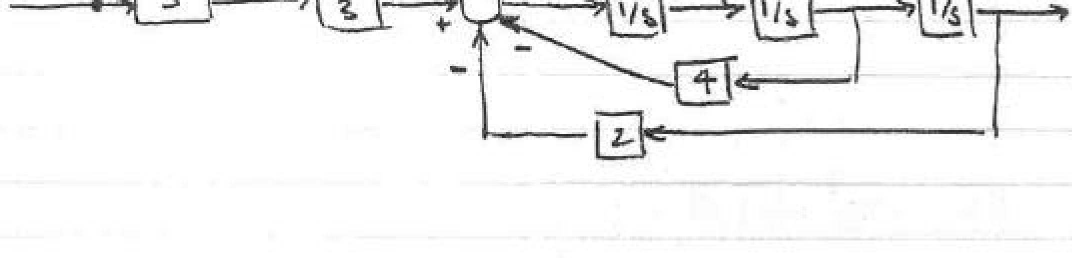
(b)



c)



d)



HW11-5.

In Problems 13.24 through 13.26, determine the closed-loop transfer function $Y(s)/U(s)$ as a rational function of s for the block diagram shown in the figure cited.

* 13.24 Figure P13.24.

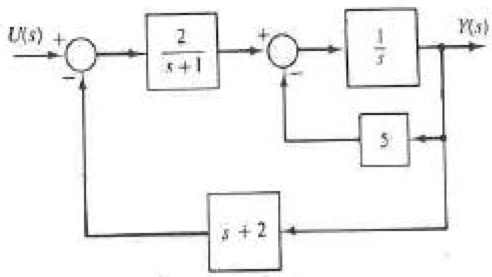


FIGURE P13.24

Inner loop with feed back

$$T_1 = \frac{1}{s}\frac{1}{\left(1+\frac{5}{s}\right)} = \frac{1}{s+5}$$

Elements in series

$$T_2 = \frac{2}{s+1}\cdot\frac{1}{s+5} = \frac{2}{(s+1)(s+5)}$$

Outer Loop

$$T = \frac{T_2}{1+T_2\cdot(s+2)} = \frac{2}{(s+1)(s+5)\left(1+\frac{(s+2)2}{(s+1)(s+5)}\right)}$$

$$T = \frac{2}{(s+1)(s+5)+2(s+2)} = \frac{2}{s^2+6s+5+2s+4}$$

$$T = \frac{2}{s^2+8s+9}$$

13.25 Figure P13.25.

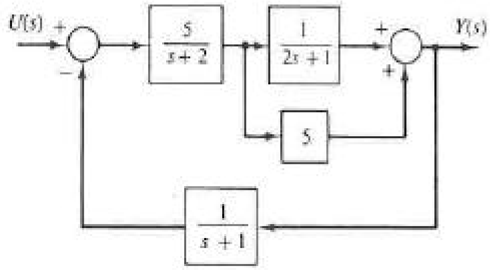
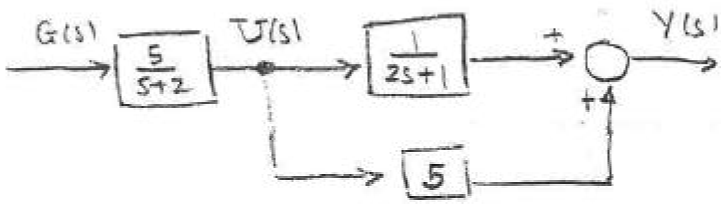


FIGURE P13.25

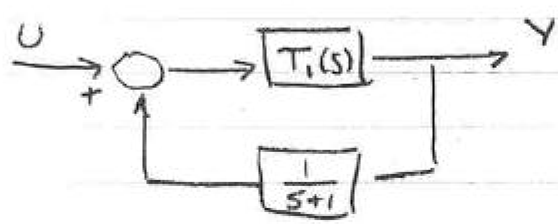
Equivalent transfer function for upper loop



$$Y(s) = U(s)\cdot\left(5 + \frac{1}{2s+1}\right) = \frac{10s+5+1}{2s+1} = \frac{10s+6}{2s+1}$$

$$\frac{Y(s)}{G(s)} = \frac{5}{s+2}\cdot\frac{10s+6}{2s+1} = \frac{5(10s+6)}{(s+2)(2s+1)} \triangleq T_1(s)$$

Thus, block diagram look like



(use standard result for closed loop)

$$\Rightarrow T = \frac{T_1}{1+T_1\cdot\frac{1}{s+1}} = \frac{5(10s+6)}{(s+2)(2s+1)\left(1+\frac{5(10s+6)}{(s+2)(2s+1)(s+1)}\right)}$$

$$T = \frac{5(10s+6)(s+1)}{(s+2)(2s+1)(s+1)+5(10s+6)} = \frac{50s^2+80s+30}{2s^3+8s^2+57s+32}$$

13.33 a) Find the closed-loop transfer function $T(s) = Y(s)/U(s)$ as a ratio of polynomials for the block diagram shown in Figure P13.33.
b) Express the damping ratio ζ and the undamped natural frequency ω_n in terms of K. Show that both poles of $T(s)$ are on the negative real axis for $0 \le K \le 1/4$.

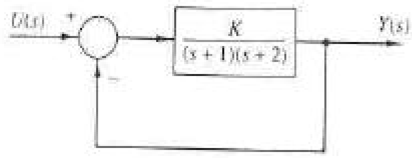


FIGURE P13.33

a). $$T = \frac{G}{1+G} = \frac{K}{(s+1)(s+2)\left[1 + \frac{K}{(s+1)(s+2)}\right]}$$

$$T = \frac{K}{(s+1)(s+2)+K} = \frac{K}{s^2+3s+(2+K)}$$

By analogy with mass / spring system. For mass/spring system

$$T = \frac{1}{ms^2+Bs+K_s}$$

K_s - spring stiffness
m - mass
B - damping.

Natural frequency $= \omega_0^2 = \frac{K_s}{m}$

Damping ratio $= \zeta = \frac{B}{2\sqrt{K_s m}}$

In this case: $m = 1$, $B = 3$, $K_s = 2+K$

$\therefore \quad \omega_0 = \sqrt{2+K}, \quad \zeta = \frac{3}{2\sqrt{2+K}}$

Poles are on the negative real axis if $\zeta \ge 1$

$$\Rightarrow \quad \frac{3}{2\sqrt{2+K}} \ge 1 \quad \Rightarrow \quad \sqrt{K+2} \le \frac{3}{2}$$

$$\Rightarrow \quad K+2 \le \frac{9}{4} \quad \Rightarrow \quad K \le \frac{1}{4}$$

Extra Credit. (25%).

a. From (32), page 406 of text

$$\ddot{\omega} + (1)\,\dot{\omega} + \frac{1}{1}\,\omega = \frac{1}{1}\,e_i(t)$$

or

$$\ddot{\omega} + \dot{\omega} + \omega = e_i(t)$$

Take Laplace T.

$$\Omega(s) = \frac{E_i(s)}{s^2+s+1} = \frac{100}{s(s^2+s+1)}$$

$$\Omega(s) = \frac{A}{s} + F(s)$$

$$A = \lim_{s \to 0} s\,\Omega(s) = 100$$

$$F(s) = \frac{\cancel{100} - 100s^2 - 100s - \cancel{100}}{s(s^2+s+1)} = \frac{-100\,(s+1)}{s^2+s+1}$$

Complete square in denominator

$$F(s) = -100 \cdot \frac{s+1}{(s+\frac{1}{2})^2 + 0.75}$$

Standard L.T. - see page 646 - has form

$$F(s) = -100 \cdot \frac{Bs + C}{(s+a)^2 + \omega^2}$$

where $B = 1$, $C = 1$, $a = \frac{1}{2}$ $\omega = \sqrt{.75}$

From Table:

$$\omega = 100\left[1 - e^{-\frac{t}{2}}\left(\cos\omega t + \frac{(1/2)}{\omega}\sin\omega t\right)\right]$$

See MathCad Plot on next page.

b. 5% OS $\Rightarrow$ $e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = 0.05$

Take natural log of both sides $\Rightarrow$ $-\frac{\zeta\pi}{\sqrt{1-\zeta^2}} = -2.996$

Divide by π & square both sides

$\Rightarrow$

$$\frac{\zeta^2}{1-\zeta^2} = 0.909 \Rightarrow \zeta^2 = \frac{0.909}{1.909}$$

$$\zeta = 0.69$$

From (32) if $R_A \neq 1$, we get. (page 406)

$$\ddot{\omega} + R_A\dot{\omega} + 1\omega = e_i(t)$$

$\therefore$ R_A is analogous to B for a mechanical system $K = 1$, $m = 1$.

$$\therefore \zeta = \frac{B}{2\sqrt{Km}} = \frac{B}{2} = \frac{R_A}{2}$$

$$\Rightarrow R_A = 2\cdot(0.69) = 1.38$$

c. From page 406, equation (32)

$$\ddot{\omega} + \dot{\omega} + \omega = e_i(t) - \dot{\tau}_L - \tau_L(t)$$

For steady state $\dot{\omega} = \ddot{\omega} = \dot{\tau}_L = 0$

$$\therefore \quad \omega = 100 - 20 = 80 \text{ rad/sec.}$$