HW10 - 1.

10.20 The rotor shown in Figure P10.20 is driven by a torque $\tau_a(t)$. The rotor has a moment of inertia J, but the friction is negligible. The field winding is excited by a constant voltage source, resulting in a constant magnetic flux. The armature resistance is denoted by R_A, and the armature inductance is negligible. Use (29) to obtain expressions for e_m and τ_e.

a) With the rotor initially at rest and with the switch in the left-hand position connecting the armature to a short circuit, the applied torque is $\tau_a(t) = U(t)$. Find and sketch ω as a function of time.

b) After the rotor has reached a steady-state speed under the conditions of part (a), the switch is thrown to the right, thereby connecting the armature to the battery. Find and sketch ω versus t if a constant unit torque continues to be applied to the rotor.

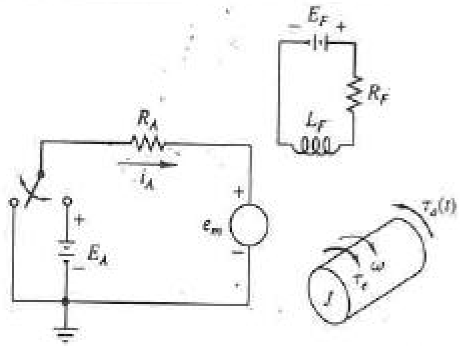


FIGURE P10.20

c) After a new steady-state speed has been reached under the conditions of part (b), the applied torque is removed, with the switch still in the right-hand position. Again find and sketch ω versus t.

Refer to equation (29)

$$\tau_e = \alpha i_A \quad (1)$$

$$e_m = \alpha \omega \quad (2)$$

a) Switch in left hand position

Equation for armature circuit is:

$$R_A i_A + e_m = 0 \quad \text{or using (2)}$$

$$R_A i_A + \alpha \omega = 0 \quad (3)$$

The equation of motion for the rotor is

$$J\dot{\omega} = +\tau_e - \tau_a \qquad (4)$$

or using (1)

$$J\dot{\omega} - \alpha i_A = -\tau_a \qquad (5)$$

$$(3) \Rightarrow \qquad i_A = -\frac{\alpha\omega}{R_A}$$

$$J\dot{\omega} + \frac{\alpha^2\omega}{R_A} = -\tau_A \qquad (6)$$

Assume $\tau_A = U(t)$, $\omega(0) = 0$

Take Laplace T.

$$J s\Omega(s) + \frac{\alpha^2}{R_A}\Omega(s) = -\frac{1}{s}$$

$$\Omega(s) = -\frac{1}{s}\frac{1}{J} \cdot \frac{1}{s + \frac{\alpha^2}{R_A J}} \qquad (7)$$

Use Partial fractions

$$\Omega(s) = \frac{A}{s} + \frac{B}{s + \frac{\alpha^2}{R_A J}}$$

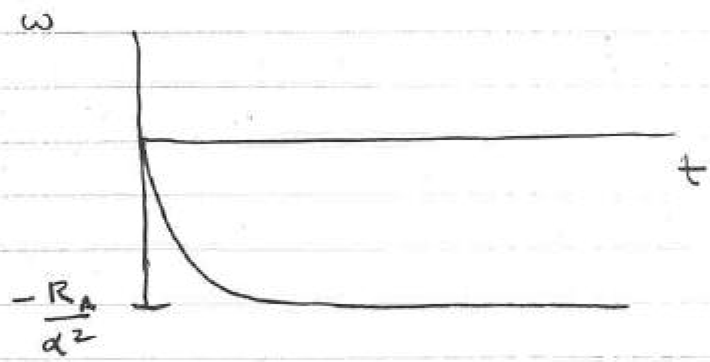
$$A = -\frac{1}{J} \cdot \frac{1}{\frac{\alpha^2}{R_A J}} = -\frac{R_A}{\alpha^2}$$

$$B = +\frac{1}{\cancel{J}\frac{\alpha^2}{R_A \cancel{J}}} = \frac{R_A}{\alpha^2}$$

$$\therefore \ \Omega(s) = \frac{R_A}{\alpha^2}\left(-\frac{1}{s} + \frac{1}{s + \frac{\alpha^2}{R_A J}} \right) \qquad (8)$$

$$\therefore \ \omega(t) = \frac{R_A}{\alpha^2}\left(e^{-\frac{\alpha^2 t}{R_A J}} - 1 \right) \qquad (9)$$

for $t > 0$



b) For steady state $\omega = -\frac{R_A}{\alpha^2}$

With the switch thrown the equation for the armature circuit becomes

$$-E_A + R_A i_A + e_m = 0$$

or $-E_A + R_A i_A + \alpha\omega = 0 \Rightarrow i_A = -\frac{\alpha\omega + E_A}{R_A}$

(5) becomes

$$J\dot{\omega} + \alpha\left(\frac{\alpha\omega - E_A}{R_A}\right) = -\tau_A$$

$$J\dot{\omega} + \frac{\alpha^2\omega}{R_A} = -\tau_A + \frac{\alpha E_A}{R_A} \qquad (10)$$

Can solve with either L.T. or ODE theory

ODE Theory. Suppose we restart our stop watch at the time we close the circuit.

Then $\omega(0) = -\dfrac{R_A}{\alpha^2}$

The solution to (10) is $(\tau_A = U(t) = 1 \text{ for } t>0)$

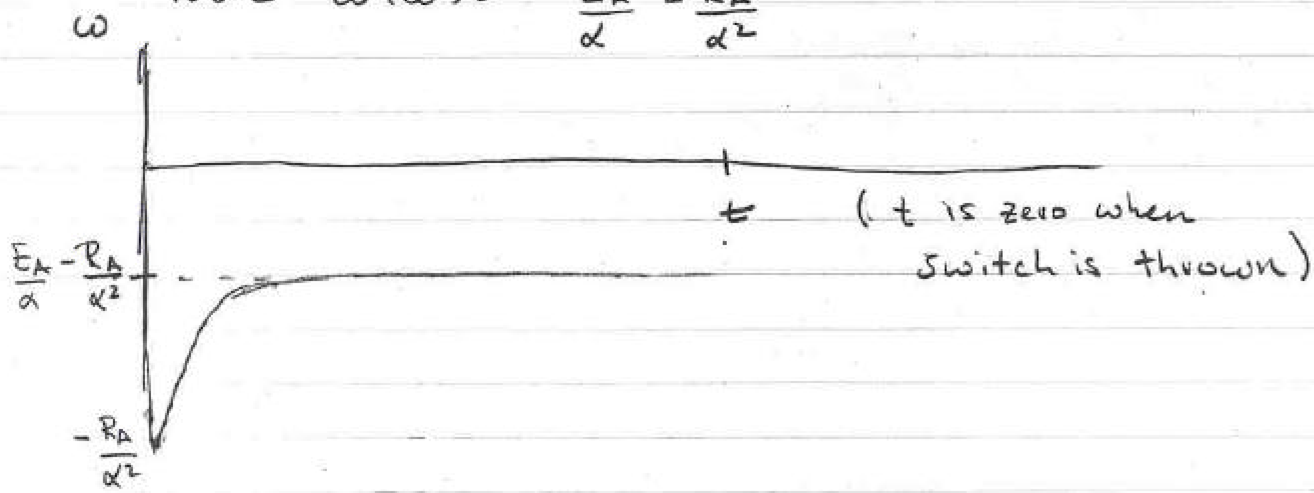
$$\omega_h = A\, e^{-\frac{\alpha^2 t}{R_A J}} \qquad \omega_p = \left(-1 + \frac{\alpha E_A}{R_A}\right)\frac{R_A}{\alpha^2}$$

Apply I.C's $\Rightarrow$

$$A - \cancel{\frac{R_A}{\alpha^2}} + \frac{E_A}{\alpha} = -\cancel{\frac{R_A}{\alpha^2}} \quad \Rightarrow \quad A = -\frac{E_A}{\alpha}$$

$$\therefore\ \omega(t) = \frac{E_A}{\alpha}\left(1 - e^{-\frac{\alpha^2 t}{R_A J}}\right) - \frac{R_A}{\alpha^2}$$

Note $\omega(\infty) = \dfrac{E_A}{\alpha} - \dfrac{R_A}{\alpha^2}$



c) In this case we re-set our stop watches so that $t=0$ corresponds to when we remove the external torque. Thus, $\omega(0) = \dfrac{E_A}{\alpha} - \dfrac{R_A}{\alpha^2}$

The ODE is

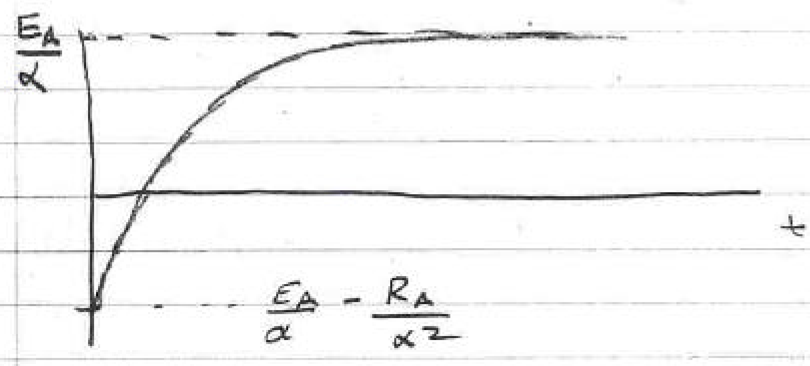
$$J\dot{\omega} + \frac{\alpha^2 \omega}{R_A} = \frac{\alpha E_A}{R_A}$$

The solution is (the same as part b)

$$\omega = \omega_p + \omega_h = \frac{E_A}{\alpha} + B e^{-\frac{\alpha^2 t}{R_A J}}$$

$$\omega(0) = \frac{E_A}{\alpha} - \frac{R_A}{\alpha^2} = \frac{E_A}{\alpha} + B \Rightarrow B = -\frac{R_A}{\alpha^2}$$

$$\therefore \quad \omega = \frac{E_A}{\alpha} - \frac{R_A}{\alpha^2} e^{-\frac{\alpha^2 t}{R_A J}}$$



(it is zero when the load is removed).

* **10.21** The magnetic field of a small motor is supplied by a permanent magnet. The following two sets of measurements are made in the steady state with a 6-V battery connected across the armature. When there is no load and negligible friction, the motor rotates at 100 rad/s. When the motor is mechanically blocked to prevent its movement, the armature current is 2 A. The rotor is then attached to a propeller, which creates a viscous-friction load of 2.0×10^{-3} N·m·s, and the 6-V battery is again connected across the armature. Find the steady-state speed of the propeller.

For steady state ($L \frac{di_A}{dt} = 0$), the equation for the voltage drop in the armature $\Rightarrow$

$$-E_A + R_A i_A + \alpha\omega = 0 \qquad (1)$$

and the steady state motion for the rotor

$$J\overset{0}{\cancel{\dot\omega}} = -B\omega + i_A\omega - \tau_a = 0 \qquad (2)$$

Mechanically blocked $\Rightarrow$

$E_A = 6V \quad \omega = 0 \Rightarrow i_A = 2A \Rightarrow$

$$-6V = -R_A \cdot 2A \Rightarrow R_A = 3\Omega \qquad (3)$$

No load, no friction

The equation of motion for the rotor is

$$J\dot\omega = \tau_e = \alpha i_A \qquad (4)$$

The equation for the current is

$$-E_A + R_A i_A + \alpha\omega = 0 \Rightarrow i_A = \frac{E_A - \alpha\omega}{R_A} \qquad (5)$$

So $$J\dot{\omega} + \frac{\alpha^2 \omega}{R_A} = \frac{\alpha E_A}{R_A} \quad (6)$$

Implies steady state speed is $(\dot{\omega} = 0)$

$$\omega_{ss} = \frac{\alpha E_A}{\cancel{R_A}} \frac{\cancel{R_A}}{\alpha^2} = \frac{E_A}{\alpha} \quad (7)$$

$$\therefore \quad \alpha = \frac{E_A}{\omega_{ss}} = \frac{6V}{100 \text{ rad/sec}} = 0.06 \frac{V}{\text{rad}} \text{sec} \quad (8)$$

Attach a viscous load of $B\omega$ $(B = 2 \cdot 10^{-3} \text{ Nm}\cdot\text{s})$

Equation of motion (4) becomes

$$J\dot{\omega} = \tau_e - B\omega = \alpha i_A - B\omega \Rightarrow \quad (9)$$

$$J\dot{\omega} + B\omega = \alpha\left(\frac{E_A - \alpha\omega}{R_A}\right) \quad (10)$$

or

$$J\dot{\omega} + \left(B + \frac{\alpha^2}{R_A}\right)\omega = \frac{\alpha E_A}{R_A} \quad (11)$$

Steady-state $\Rightarrow$ $(\dot{\omega} = 0)$

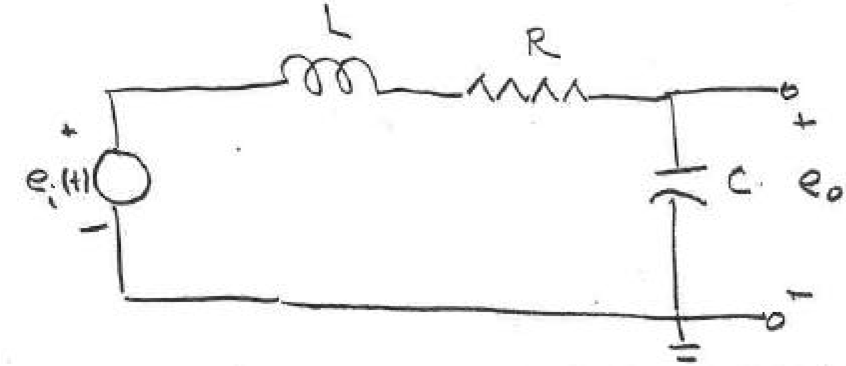
$$\omega_{ss} = \frac{\alpha E_A}{\cancel{R_A}\left(\frac{B R_A + \alpha^2}{\cancel{R_A}}\right)}$$

$$\omega_{ss} = \frac{6 \cdot 6}{100\left(\frac{2}{1000} \cdot 3 + \left(\frac{6}{100}\right)^2\right)}$$

$$\omega_{ss} = \frac{36}{\left(\frac{6}{10} + \frac{36}{100}\right)} = \frac{36}{.96} = 37.5 \frac{\text{rad}}{\text{sec}}$$

HW 10-8

1. Suppose you have the circuit shown below.
   a. What is its impedance?
   b. What is the transfer function, i.e. the ratio $E_0(s)/E_i(s)$?
   c. Suppose L = 1 and C = 1. Suppose $e_i(t) = H(t)$ where H is the Heaviside step function.
      i. What does the output voltage look like as a function of time if R = 1?
      ii. What value of resistance would result in an overshoot of 10%? Plot the response as a function of time for this value of R. What is the response time t_R, i.e. the time at which $e_o(t)$ first equals 90% of its steady state value?
      iii. Repeat part ii for an overshoot of 2%. (The response time is the time at which it equals 98% of its steady-state value).
      iv. How much slower is the system's response, i.e. what is the percentage increase in the response time when the overshoot is reduced from 10% to 2%?
      v. How does your answer to iv depend on the values of L or C?



a. Impedance: elements in series ⇒ add impedances.

$$Z = Ls + R + \frac{1}{Cs}$$

b. $I \cdot Z = E_i$

$$E_o = \frac{1}{Cs} \cdot I(s) = \frac{E_i}{Cs \cdot Z}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{Cs \cdot Z} = \frac{1}{LCs^2 + RCs + 1}$$

c. Given $L=1$, $C=1$. Laplace T. for output looks like

$$E_o(s) = \frac{1}{s^2 + Rc + 1} E_i(s) = \frac{1}{s}\frac{1}{(s^2+Rs+1)}$$

This is the problem that we solved in class where

Use solution from class notes or use the following:

Partial Fractions

$$E_o = \frac{A}{s} + F(s)$$

$$A = \lim_{s \to 0} s\,E_o(s) = \cdot 1$$

$$F(s) = \frac{1}{s}\frac{1}{(s^2+Rs+1)} - \frac{1}{s} = \frac{\cancel{1} - s^2 - Rs - \cancel{1}}{s\,(s^2+Rs+1)}$$

$$= -\frac{(s+R)}{s^2+Rs+1}$$

Complete the square for denominator.

$$F(s) = -\frac{s+R}{\left(s+\frac{R}{2}\right)^2 + 1 - \frac{R^2}{4}}$$

(see Table B.1) p 646

$$= \frac{Bs + C}{(s+a)^2 + \omega^2}$$

$B = -1 \quad C = -R$

$a = \frac{R}{2} \quad \omega = \sqrt{1 - \frac{R^2}{4}}$

$$\therefore \quad e_o(t) = 1 - e^{-\frac{R}{2}t}\left[\cos\omega t + \left(\frac{R - \frac{R}{2}}{\omega}\right)\sin\omega t\right]$$

$$= 1 - e^{-\frac{R}{2}t}\left[\cos\omega t + \frac{R/2}{\sqrt{1 - R^2/4}}\sin\omega t\right]$$

i) See Mathcad plot of $e_o(t)$ for $R = 1$.

ii) From class

$$OS = e^{-\frac{\zeta}{\sqrt{1-\zeta^2}}\cdot\pi} = .1$$

$$\Rightarrow \quad \frac{-\zeta}{\sqrt{1-\zeta^2}} = \frac{-2.302}{\pi} \Rightarrow \frac{\zeta}{\sqrt{1-\zeta^2}} = 0.733$$

$$\Rightarrow \quad \frac{\zeta^2}{1-\zeta^2} = (.733)^2 = 0.537$$

$$\Rightarrow \quad \zeta^2 = 0.537 - \zeta^2(0.537)$$

$$\Rightarrow \quad \zeta^2 = \frac{0.537}{1 + 0.537} = 0.3495$$

$$\zeta = 0.591$$

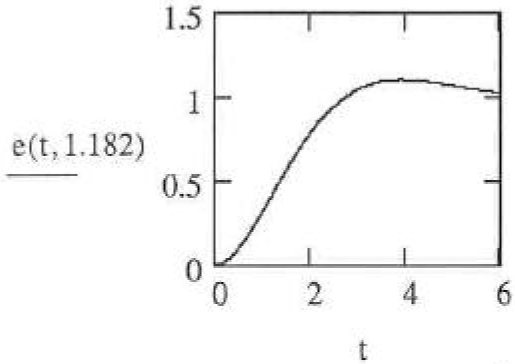
In this problem $\zeta = \frac{R}{2\sqrt{L\cdot\frac{1}{C}}} = \frac{R}{2} \Rightarrow R = 1.182$

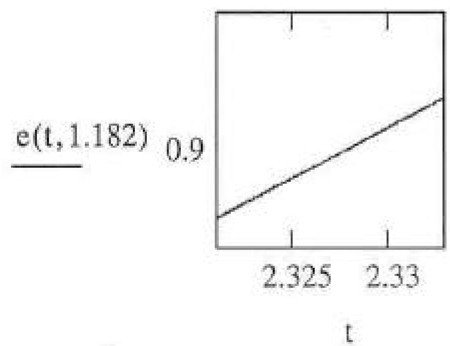
See Math Cad Plot.

Define Response

$$w(R) := \sqrt{1-\left(\frac{R}{2}\right)^2}$$

$$e(t,R) := 1-\left(e^{-R\cdot\frac{t}{2}}\right)\cdot\left[\cos(w(R)\cdot t)+\left[\frac{R}{2\cdot\sqrt{1-\left(\frac{R}{2}\right)^2}}\right]\cdot\sin(w(R)\cdot t)\right]$$

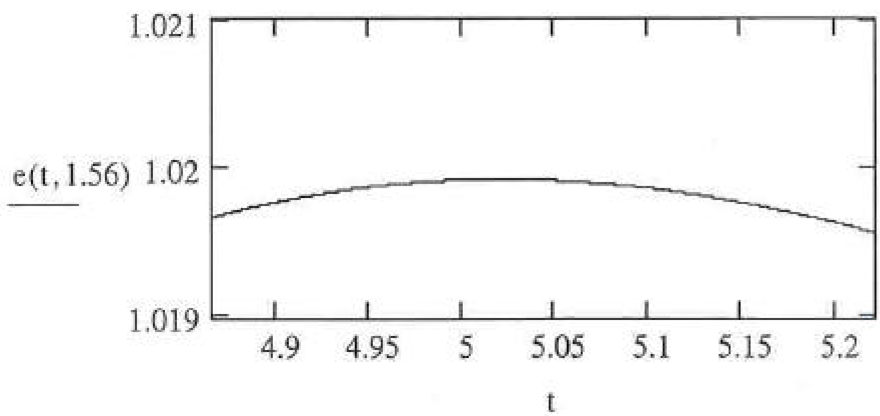




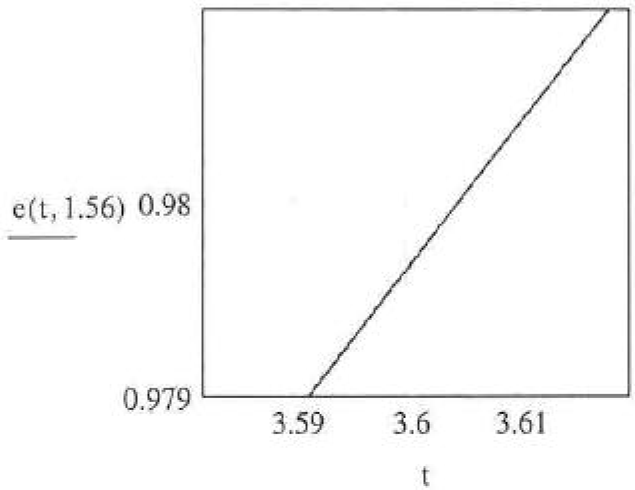
Response time is approximately 2.33 seconds

Try zeta for 2 % overshoot.

Zoom in on Maximum - Note this checks



Find Response Time



Response time is about 3.6 seconds

Percent increase is about 54% $\left(\frac{3.6 - 2.33}{2.33}\right) \cdot 100 = 54.506$