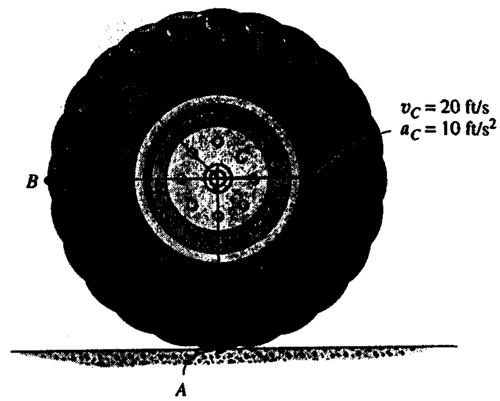
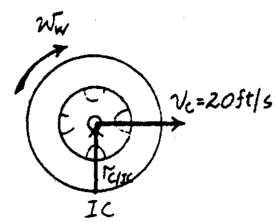
Homework #8 Solutions

***16-109.** At the instant shown, the center of the wheel moves to the right with a velocity of 20 ft/s and has an acceleration of 10 ft/s^2. Determine the acceleration of points A and B at this instant. Assume that the wheel does not slip at A.



Velocity Analysis : The angular velocity of the wheel can be obtained by using the method of instantaneous center of zero velocity. Since the wheel rotates without slipping about point A, i.e: $v_A = 0$, then point A is the location of the instantaneous center.

$$v_C = \omega_W r_{C/IC}$$
$$20 = \omega_W \left(\frac{10}{12}\right)$$
$$\omega_W = 24.0 \text{ rad/s}$$



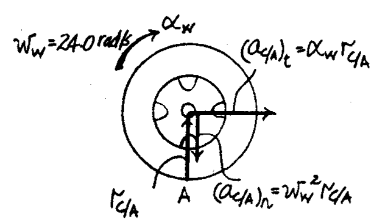
Acceleration Equation : Angular acceleration of the wheel can be obtain by analyzing the motion of points A and C. Since point a is at rest, $(a_A)_t = 0$. Applying Eq. 16 – 17, we have

$$\mathbf{a}_C = \mathbf{a}_A + (\mathbf{a}_{C/A})_t + (\mathbf{a}_{C/A})_n$$

$$\left[\underset{\rightarrow}{10}\right] = \left[\underset{\uparrow}{a_A}\right] + \left[\underset{\rightarrow}{\alpha_W\left(\frac{10}{12}\right)}\right] + \left[\underset{\downarrow}{24.0^2\left(\frac{10}{12}\right)}\right]$$

$$(\overset{+}{\rightarrow}) \qquad 10 = \alpha_W\left(\frac{10}{12}\right) \qquad \alpha_W = 12.0 \text{ rad/s}^2$$

$$(+\uparrow) \qquad 0 = a_A - 24.0^2\left(\frac{10}{12}\right) \qquad a_A = 480 \text{ ft/s}^2 \uparrow \qquad \textbf{Ans}$$



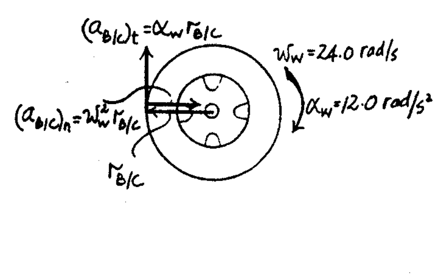
The acceleration of point B can be obtain by analyzing the motion of points B and C. Applying Eq. 16 – 17, we have

$$\mathbf{a}_B = \mathbf{a}_C + (\mathbf{a}_{B/C})_t + (\mathbf{a}_{B/C})_n$$

$$\left[\underset{\rightarrow}{(a_B)_x}\right] + \left[\underset{\uparrow}{(a_B)_y}\right] = \left[\underset{\rightarrow}{10}\right] + \left[\underset{\uparrow}{12.0\left(\frac{10}{12}\right)}\right] + \left[\underset{\rightarrow}{24.0^2\left(\frac{10}{12}\right)}\right]$$

$$(\overset{+}{\rightarrow}) \qquad (a_B)_x = 10 + 24.0^2\left(\frac{10}{12}\right) = 490 \text{ ft/s}^2 \rightarrow$$

$$(+\uparrow) \qquad (a_B)_y = 12.0\left(\frac{10}{12}\right) = 10.0 \text{ ft/s}^2 \uparrow$$

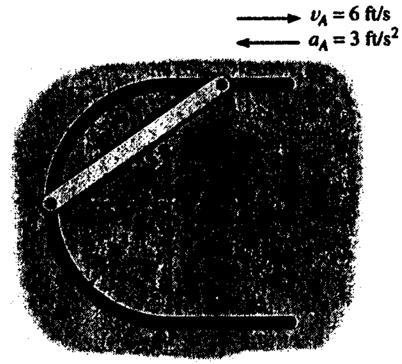


Thus, the magnitude and directional angle of $\mathbf{a}_B$ are

$$a_B = \sqrt{(a_B)_x^2 + (a_B)_y^2} = \sqrt{490^2 + 10.0^2} = 490 \text{ ft/s}^2 \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\frac{(a_C)_y}{(a_C)_x} = \tan^{-1}\frac{10.0}{490} = 1.17^\circ \qquad \textbf{Ans}$$

16-111. The rod is confined to move along the path due to the pins at its ends. At the instant shown, point A has the motion shown. Determine the velocity and acceleration of point B at this instant.



$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$v_B\mathbf{j} = 6\mathbf{i} + (-\omega\mathbf{k}) \times (-4\mathbf{i} - 3\mathbf{j})$$

$$0 = 6 - 3\omega; \qquad \omega = 2 \text{ rad/s}$$

$$v_B = 4\omega = 4(2) = 8 \text{ ft/s} \uparrow \qquad \textbf{Ans}$$

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

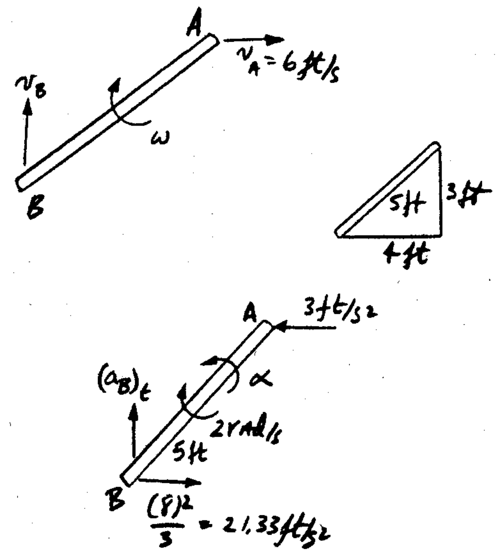
$$21.33\mathbf{i} + (a_B)_t\mathbf{j} = -3\mathbf{i} + \alpha\mathbf{k} \times (-4\mathbf{i} - 3\mathbf{j}) - (-2)^2(-4\mathbf{i} - 3\mathbf{j})$$

$$(\overset{+}{\rightarrow}) \qquad 21.33 = -3 + 3\alpha + 16; \qquad \alpha = 2.778 \text{ rad/s}^2$$

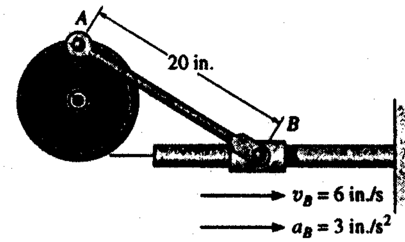
$$(+\uparrow) \qquad (a_B)_t = -(2.778)(4) + 12 = 0.8889 \text{ ft/s}^2$$

$$a_B = \sqrt{(21.33)^2 + (0.8889)^2} = 21.4 \text{ ft/s}^2 \qquad \textbf{Ans}$$

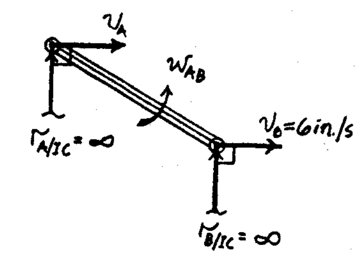
$$\theta = \tan^{-1}\left(\frac{0.8889}{21.33}\right) = 2.39^\circ \angle \qquad \textbf{Ans}$$



16-118. At a given instant, the slider block B is traveling to the right with the velocity and acceleration shown. Determine the angular acceleration of the wheel at this instant.



Velocity Analysis : The angular velocity of link AB can be obtained by using the method of instantaneous center of zero velocity. Since $\mathbf{v}_A$ and $\mathbf{v}_B$ are parallel, $r_{A/IC} = r_{B/IC} = \infty$. Thus, $\omega_{AB} = 0$. Since $\omega_{AB} = 0$, $v_A = v_B = 6$ in./s. Thus, the angular velocity of the wheel is $\omega_W = \dfrac{v_A}{r_{OA}} = \dfrac{6}{5} = 1.20$ rad/s.



Acceleration Equation : The acceleration of point A can be obtained by analyzing the angular motion of link OA about point O. Here, $\mathbf{r}_{OA} = \{5\mathbf{j}\}$ in.

$$\begin{aligned} \mathbf{a}_A &= \boldsymbol{\alpha}_W \times \mathbf{r}_{OA} - \omega_W^2 \mathbf{r}_{OA} \\ &= (-\alpha_W \mathbf{k}) \times (5\mathbf{j}) - 1.20^2(5\mathbf{j}) \\ &= \{5\alpha_W \mathbf{i} - 7.20\mathbf{j}\} \text{ in./s}^2 \end{aligned}$$

Link AB is subjected to general plane motion. Applying Eq. 16 – 18 with $\mathbf{r}_{B/A} = \{20\cos 30°\mathbf{i} - 20\sin 30°\mathbf{j}\}$ in. $= \{17.32\mathbf{i} - 10.0\mathbf{j}\}$ in., we have

$$\begin{aligned} \mathbf{a}_B &= \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} \\ 3\mathbf{i} &= 5\alpha_W \mathbf{i} - 7.20\mathbf{j} + \alpha_{AB}\mathbf{k} \times (17.32\mathbf{i} - 10.0\mathbf{j}) - \mathbf{0} \\ 3\mathbf{i} &= (10.0\alpha_{AB} + 5\alpha_W)\mathbf{i} + (17.32\alpha_{AB} - 7.20)\mathbf{j} \end{aligned}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ component, we have

$$3 = 10.0\alpha_{AB} + 5\alpha_W \qquad [1]$$

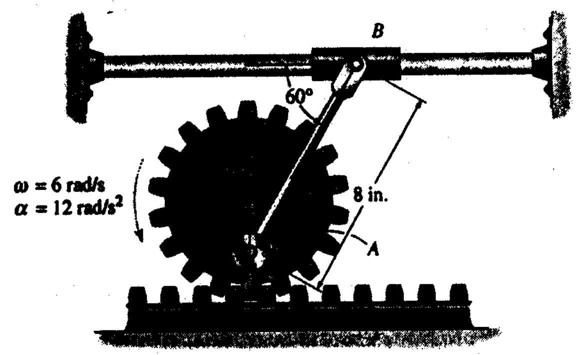
$$0 = 17.32\alpha_{AB} - 7.20 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$\alpha_{AB} = 0.4157 \text{ rad/s}^2$$

$$\alpha_W = -0.2314 \text{ rad/s}^2 = 0.231 \text{ rad/s}^2 \qquad \textbf{Ans}$$

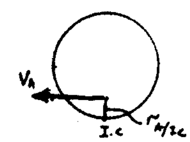
16-126. At a given instant, the gear has the angular motion shown. Determine the accelerations of points A and B on the link and the link's angular acceleration at this instant.



For the gear

$v_A = \omega r_{A/IC} = 6(1) = 6$ in./s

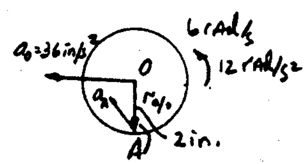
$\mathbf{a}_O = -12(3)\mathbf{i} = \{-36\mathbf{i}\}$ in./s² $\mathbf{r}_{A/O} = \{-2\mathbf{j}\}$ in. $\alpha = \{12\mathbf{k}\}$ rad/s²



$\mathbf{a}_A = \mathbf{a}_O + \alpha \times \mathbf{r}_{A/O} - \omega^2 \mathbf{r}_{A/O}$

$= -36\mathbf{i} + (12\mathbf{k}) \times (-2\mathbf{j}) - (6)^2(-2\mathbf{j})$

$= \{-12\mathbf{i} + 72\mathbf{j}\}$ in./s²

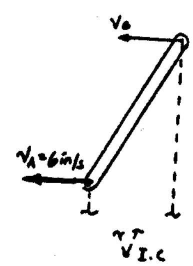


$a_A = \sqrt{(-12)^2 + 72^2} = 73.0$ in./s²   **Ans**

$\theta = \tan^{-1}\left(\frac{72}{12}\right) = 80.5°$ ↘   **Ans**

For link AB

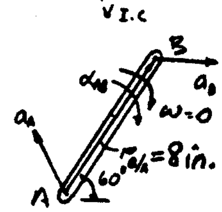
The IC is at ∞, so $\omega_{AB} = 0$, i.e.,



$\omega_{AB} = \frac{v_A}{r_{A/IC}} = \frac{6}{\infty} = 0$

$\mathbf{a}_B = a_B\mathbf{i}$ $\alpha_{AB} = -\alpha_{AB}\mathbf{k}$ $\mathbf{r}_{B/A} = \{8\cos 60°\mathbf{i} + 8\sin 60°\mathbf{j}\}$ in.

$\mathbf{a}_B = \mathbf{a}_A + \alpha_{AB} \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$

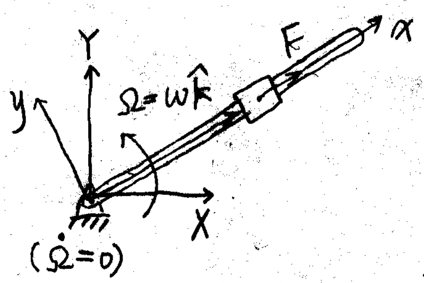


$a_B\mathbf{i} = (-12\mathbf{i} + 72\mathbf{j}) + (-\alpha_{AB}\mathbf{k}) \times (8\cos 60°\mathbf{i} + 8\sin 60°\mathbf{j}) - \mathbf{0}$

$(\overset{+}{\rightarrow})$ $a_B = -12 + 8\sin 60°(18) = 113$ in./s² →   **Ans**

$(+\uparrow)$ $0 = 72 - 8\cos 60°\alpha_{AB}$ $\alpha_{AB} = 18$ rad/s² ↻   **Ans**

5.



$$\vec{a}_{XYZ} = \vec{a}_{xyz} + 2\vec{\Omega}\times\vec{v}_{xyz} + \vec{\Omega}\times(\vec{\Omega}\times\vec{r})$$

$$\vec{a}_{xyz} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

$$2\vec{\Omega}\times\vec{v}_{xyz} = 2\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ \dot{x} & \dot{y} & 0 \end{vmatrix} = 2(-\omega\dot{y}\hat{i} + \omega\dot{x}\hat{j})$$

$$\vec{\Omega}\times\vec{\Omega}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ -\omega y & \Omega x & 0 \end{vmatrix} = -\omega^2 x\hat{i} - \omega^2 y\hat{j} = -\omega^2\vec{r}$$

$$\vec{a}_{XYZ} = (\ddot{x} - 2\omega\dot{y} - \omega^2 x)\hat{i} + (\ddot{y} + 2\omega\dot{x} - \omega^2 y)\hat{j}$$

$$m\vec{a}_{XYZ} = \vec{F} \Rightarrow \begin{cases} m(\ddot{x} - 2\omega\dot{y} - \omega^2 x) = F_x \\ m(\ddot{y} + 2\omega\dot{x} - \omega^2 y) = F_y \end{cases}$$

$y = 0$ in our problem $\Rightarrow m(\ddot{x} - \omega^2 x) = F_x = F$

$$\ddot{x} - \omega^2 x = 0$$

$$\Rightarrow x = A\cosh\omega t + B\sinh\omega t$$

Since $\left.\begin{array}{l} x(0) = A = x_0 \\ \dot{x}(0) = \omega B = 0 \end{array}\right\} \Rightarrow \boxed{x = x_0\cosh\omega t}$

The collar will leave far and far away from the hinge.