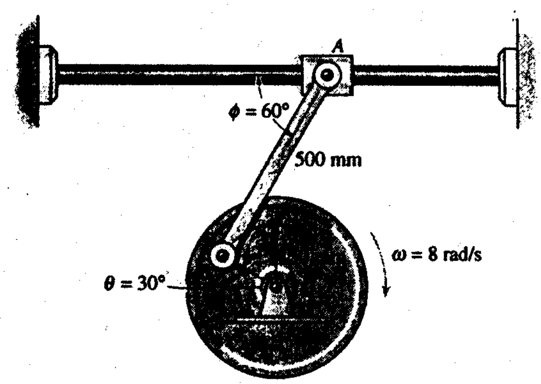
Homework #6 Solutions

16-51. The wheel is rotating with an angular velocity $\omega = 8$ rad/s. Determine the velocity of the collar A at the instant $\theta = 30°$ and $\phi = 60°$. Also, sketch the location of bar AB when $\theta = 0°, 30°$, and $60°$ to show its general plane motion.



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$\underset{\rightarrow}{v_A} = \underset{\angle 60°}{1.2} + \underset{\searrow 30°}{0.5\omega_{AB}}$$

$\overset{+}{\rightarrow}$ $\quad v_A = 1.2\cos 60° + 0.5\omega_{AB}\cos 30°$

$+\uparrow$ $\quad 0 = 1.2\sin 60° - 0.5\omega_{AB}\sin 30°$

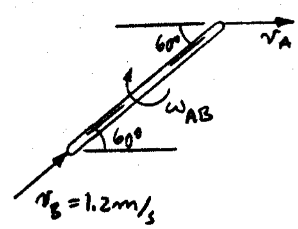
$$\omega_{AB} = 4.16 \text{ rad/s}$$

$$v_A = 2.40 \text{ m/s} \rightarrow \qquad \textbf{Ans}$$

Also, $\quad \mathbf{v}_B = \boldsymbol{\omega}\times\mathbf{r}_B$

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega}_{AB}\times\mathbf{r}_{A/B}$$

$$v_A\mathbf{i} = (-8\mathbf{k})\times(-0.15\cos 30°\mathbf{i} + 0.15\sin 30°\mathbf{j}) + (-\omega_{AB}\mathbf{k})\times(0.5\cos 60°\mathbf{i} + 0.5\sin 60°\mathbf{j})$$



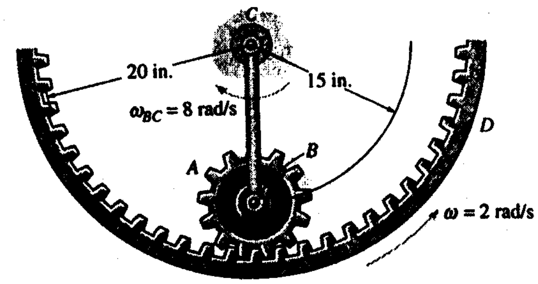
$$v_A = 0.60 + 0.433\omega_{AB}$$

$$0 = 1.039 - 0.25\omega_{AB}$$

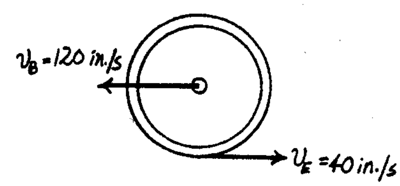
$$\omega_{AB} = 4.16 \text{ rad/s}$$

$$v_A = 2.40 \text{ m/s} \rightarrow \qquad \textbf{Ans}$$

16-59. The planetary gear A is pinned at B. Link BC rotates clockwise with an angular velocity of 8 rad/s, while the outer gear rack rotates counterclockwise with an angular velocity of 2 rad/s. Determine the angular velocity of gear A.



Kinematic Diagram : Since link BC is rotating about fixed point C, then $\mathbf{v}_B$ is always directed perpendicular to link BC and its magnitude is $v_B = \omega_{BC} r_{BC} = 8(15) = 120$ in./s. At the instant shown, $\mathbf{v}_B$ is directed to the left. Also, at the same instant, point E is moving to the right with a speed of $v_E = \omega_D r_{CE} = 2(20) = 40$ in./s.



Velocity Equation : Here, $v_{B/E} = \omega_A r_{B/E} = 5\omega_A$ which is directed to the left. Applying Eq. 16 – 15, we have

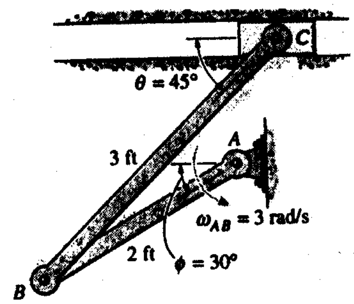
$$\mathbf{v}_B = \mathbf{v}_E + \mathbf{v}_{B/E}$$

$$\begin{bmatrix} 120 \\ \leftarrow \end{bmatrix} = \begin{bmatrix} 40 \\ \rightarrow \end{bmatrix} + \begin{bmatrix} 5\omega_A \\ \leftarrow \end{bmatrix}$$

$$(\overset{+}{\rightarrow}) \qquad -120 = 40 - 5\omega_A$$

$$\omega_A = 32.0 \text{ rad/s} \qquad \textbf{Ans}$$

16-67. If the angular velocity of link AB is $\omega_{AB} = 3$ rad/s, determine the velocity of the block at C and the angular velocity of the connecting link CB at the instant $\theta = 45°$ and $\phi = 30°$. Also, sketch the location of link BC when $\theta = 30°, 45°$, and $60°$ to show its general plane motion.



$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

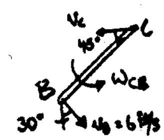
$$\begin{bmatrix} v_C \\ \leftarrow \end{bmatrix} = \begin{bmatrix} 6 \\ 30° \end{bmatrix} + \begin{bmatrix} \omega_{CB}(3) \\ 45° \end{bmatrix}$$

$(\overset{+}{\rightarrow})$ $\quad -v_C = 6\sin 30° - \omega_{CB}(3)\cos 45°$

$(+\uparrow)$ $\quad 0 = -6\cos 30° + \omega_{CB}(3)\sin 45°$

$\omega_{CB} = 2.45$ rad/s ↺ **Ans**

$v_C = 2.20$ ft/s ← **Ans**



Also,

$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$

$-v_C\mathbf{i} = (6\sin 30°\mathbf{i} - 6\cos 30°\mathbf{j}) + (\omega_{CB}\mathbf{k}) \times (3\cos 45°\mathbf{i} + 3\sin 45°\mathbf{j})$

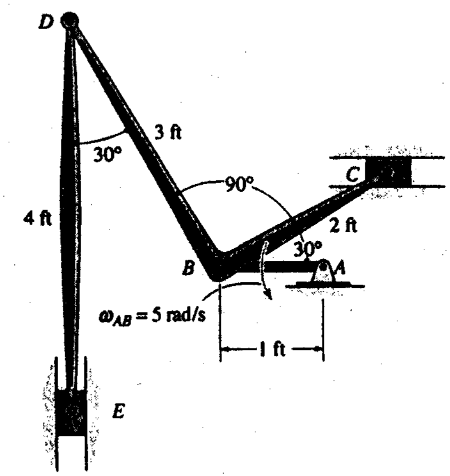
$(\overset{+}{\rightarrow})$ $\quad -v_C = 3 - 2.12\omega_{CB}$

$(+\uparrow)$ $\quad 0 = -5.196 + 2.12\omega_{CB}$

$\omega_{CB} = 2.45$ rad/s ↺ **Ans**

$v_C = 2.20$ ft/s ← **Ans**

16-74. If the link AB is rotating about the pin at A with an angular velocity of ω_{AB} = 5 rad/s, determine the velocity of blocks C and E at the instant shown.



Kinematic Diagram : Since link AB is rotating about fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link BC and its magnitude is $v_B = \omega_{AB} r_{AB} = 5(1) = 5.00$ ft/s. At the instant shown, $\mathbf{v}_B = \{-5.00\mathbf{j}\}$ ft/s. Also, block C and E are moving along the guide and directed toward *negative* x axis and *negative* y axis respectively. Then, $\mathbf{v}_C = -v_C\mathbf{i}$ and $\mathbf{v}_E = -v_E\mathbf{j}$. Since the direction of the velocity of point D are unknown, we can assume that its x and y components are directed in the *positive direction* of their respective axis.

Velocity Equation : Here, $\mathbf{r}_{C/B} = \{2\cos 30°\mathbf{i} + 2\sin 30°\mathbf{j}\}$ ft $= \{1.732\mathbf{i} + 1\mathbf{j}\}$ ft and $\mathbf{r}_{D/B} = \{-3\cos 60°\mathbf{i} + 3\sin 60°\mathbf{j}\}$ ft $= \{-1.50\mathbf{i} + 2.598\mathbf{j}\}$ ft. Applying Eq. 16 – 16 to link CBD, we have

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{CBD} \times \mathbf{r}_{C/B}$$
$$-v_C\mathbf{i} = -5.00\mathbf{j} + (\omega_{CBD}\mathbf{k}) \times (1.732\mathbf{i} + 1\mathbf{j})$$
$$-v_C\mathbf{i} = -\omega_{CBD}\mathbf{i} + (1.732\omega_{CBD} - 5.00)\mathbf{j}$$

Equating $\mathbf{i}$ and $\mathbf{j}$ components gives

$$-v_C = -\omega_{BCD} \qquad [1]$$
$$0 = 1.732\omega_{CBD} - 5.00 \qquad [2]$$

Solving Eqs. [1] and [2] yields

$$\omega_{CBD} = 2.887 \text{ rad/s}$$
$$v_C = 2.89 \text{ ft/s} \qquad \textbf{Ans}$$

The x and y component of velocity of $\mathbf{v}_D$ are given by

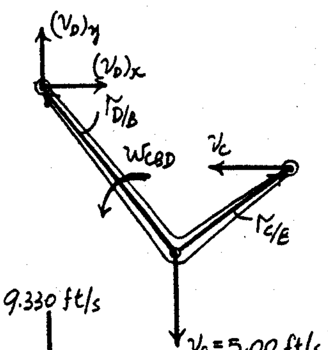
$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{CBD} \times \mathbf{r}_{D/B}$$
$$(v_D)_x\mathbf{i} + (v_D)_y\mathbf{j} = -5.00\mathbf{j} + (2.887\mathbf{k}) \times (-1.50\mathbf{i} + 2.598\mathbf{j})$$
$$(v_D)_x\mathbf{i} + (v_D)_y\mathbf{j} = -7.50\mathbf{i} - 9.330\mathbf{j}$$

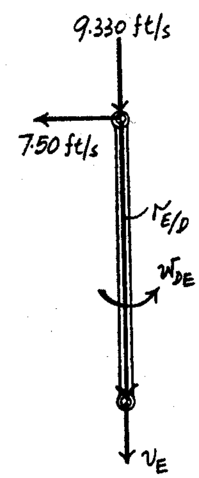
Equating $\mathbf{i}$ and $\mathbf{j}$ components gives

$$(v_D)_x = -7.50 \text{ ft/s} \qquad (v_D)_y = -9.330 \text{ ft/s}$$

Here, $\mathbf{r}_{E/D} = \{-4\mathbf{j}\}$ ft. Applying Eq. 16 – 16 to link DE, we have

$$\mathbf{v}_E = \mathbf{v}_D + \boldsymbol{\omega}_{DE} \times \mathbf{r}_{E/D}$$
$$-v_E\mathbf{j} = -7.50\mathbf{i} - 9.330\mathbf{j} + (\omega_{DE}\mathbf{k}) \times (-4\mathbf{j})$$
$$-v_E\mathbf{j} = (4\omega_{DE} - 7.50)\mathbf{i} - 9.330\mathbf{j}$$

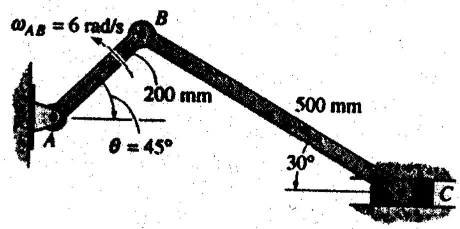




Equating $\mathbf{i}$ and $\mathbf{j}$ components gives

$$0 = 4\omega_{DE} - 7.50 \qquad \omega_{DE} = 1.875 \text{ rad/s}$$
$$v_E = 9.33 \text{ ft/s} \qquad \textbf{Ans}$$

16-86. If bar AB has an angular velocity $\omega_{AB} = 6$ rad/s, determine the velocity of the slider block C at the instant shown.



Kinematic Diagram : Since link AB is rotating about fixed point A, then $\mathbf{v}_B$ is always directed perpendicular to link AB and its magnitude is $v_B = \omega_{AB} r_{AB} = 6(0.2) = 1.20$ m/s. At the instant shown, $\mathbf{v}_B$ is directed with an angle 45° with the horizontal. Also, bock C is moving horizontally due to the constraint of the guide.

Instantaneous Center : The instantaneous center of zero velocity of bar BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sine, we have

$$\frac{r_{B/IC}}{\sin 60°} = \frac{0.5}{\sin 45°} \qquad r_{B/IC} = 0.6124 \text{ m}$$

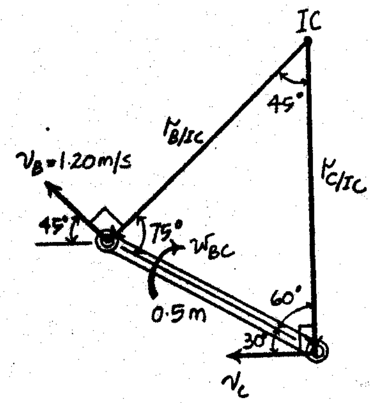
$$\frac{r_{C/IC}}{\sin 75°} = \frac{0.5}{\sin 45°} \qquad r_{C/IC} = 0.6830 \text{ m}$$

The angular velocity of bar BC is given by

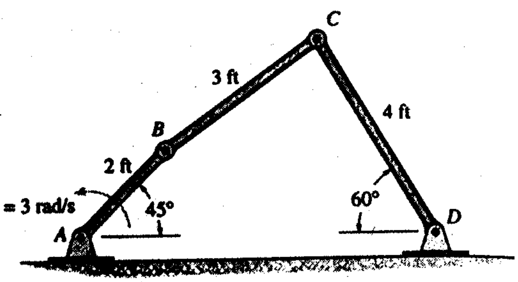
$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{1.20}{0.6124} = 1.960 \text{ rad/s}$$

Thus, the velocity of block C is

$$v_C = \omega_{BC} r_{C/IC} = 1.960(0.6830) = 1.34 \text{ m/s} \leftarrow \qquad \textbf{Ans}$$

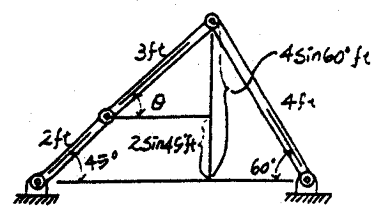


16-97. If rod AB is rotating with an angular velocity $\omega_{AB} = 3$ rad/s, determine the angular velocity of rod CD at the instant shown.



Kinematic Diagram : From the geometry, $\theta = \sin^{-1}\left(\frac{4\sin 60° - 2\sin 45°}{3}\right)$ $= 43.10°$. Since links AB and CD is rotating about fixed points A and D, then $\mathbf{v}_B$ and $\mathbf{v}_C$ are always directed perpendicular to links AB and CD respectively. The magnitude of $\mathbf{v}_B$ and $\mathbf{v}_C$ are $v_B = \omega_{AB} r_{AB} = 3(2) = 6.00$ ft/s and $v_C = \omega_{CD} r_{CD} = 4\omega_{CD}$. At the instant shown, $\mathbf{v}_B$ is directed at an angle of 45° while $\mathbf{v}_C$ is directed at 30°.

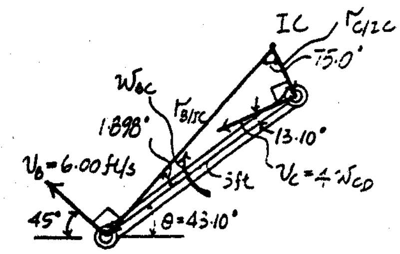
horizontal.



Instantaneous Center : The instantaneous center of zero velocity of link BC at the instant shown is located at the intersection point of extended lines drawn perpendicular from $\mathbf{v}_B$ and $\mathbf{v}_C$. Using law of sines, we have

$$\frac{r_{B/IC}}{\sin 103.1°} = \frac{3}{\sin 75°} \qquad r_{B/IC} = 3.025 \text{ ft}$$

$$\frac{r_{C/IC}}{\sin 1.898°} = \frac{3}{\sin 75°} \qquad r_{C/IC} = 0.1029 \text{ ft}$$



The angular velocity of link BC is given by

$$\omega_{BC} = \frac{v_B}{r_{B/IC}} = \frac{6.00}{3.025} = 1.983 \text{ rad/s} = 1.98 \text{ rad/s} \qquad \textbf{Ans}$$

Thus, the angular velocity of link CD is given by

$$v_C = \omega_{BC} r_{C/IC}$$
$$4\omega_{CD} = 1.983(0.1029)$$
$$\omega_{CD} = 0.0510 \text{ rad/s} \qquad \textbf{Ans}$$