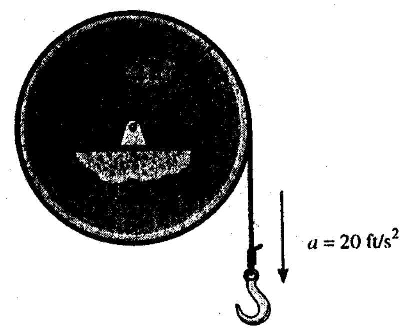
Homework #6 Solutions

16-6. The hook moves from rest with an acceleration of 20 ft/s^2. If it is attached to a cord which is wound around the drum, determine the angular acceleration of the drum and its angular velocity after the drum has completed 10 rev. How many more revolutions will the drum turn after it has first completed 10 rev and the hook continues to move downward for 4 s?



Angular Motion : The angular acceleration of the drum can be determine by applying Eq. 16 − 11.

$$a_t = \alpha r; \qquad 20 = \alpha(2) \qquad \alpha = 10.0 \text{ rad/s}^2 \qquad \textbf{Ans}$$

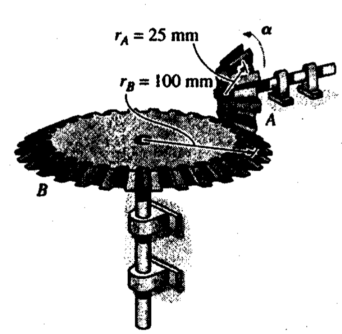
Applying Eq. 16 − 7 with $\alpha_c = \alpha = 10.0$ rad/s^2 and $\theta = (10 \text{ rev}) \times \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$ $= 20\pi$ rad, we have

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$
$$\omega^2 = 0 + 2(10.0)(20\pi - 0)$$
$$\omega = 35.45 \text{ rad/s} = 35.4 \text{ rad/s} \qquad \textbf{Ans}$$

The angular displacement of the drum 4 s after it has completed 10 revolutions can be determine by applying Eq. 16 − 6 with $\omega_0 = 35.45$ rad/s.

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$$
$$= 0 + 35.45(4) + \frac{1}{2}(10.0)(4^2)$$
$$= (221.79 \text{ rad}) \times \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 35.3 \text{ rev} \qquad \textbf{Ans}$$

16-15. Gear A is in mesh with gear B as shown. If A starts from rest and has a constant angular acceleration of α_A = 2 rad/s^2, determine the time needed for B to attain an angular velocity of ω_B = 50 rad/s.



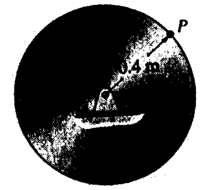
Angular Motion : The angular acceleration of gear B must be determined first. Here, $\alpha_A r_A = \alpha_B r_B$. Then,

$$\alpha_B = \frac{r_A}{r_B}\alpha_A = \left(\frac{25}{100}\right)(2) = 0.5 \text{ rad/s}^2$$

The time for gear B to attain an angular velocity of $\omega_B = 50$ rad/s can be obtained by applying Eq. 16 − 5.

$$\omega_B = (\omega_0)_B + \alpha_B t$$
$$50 = 0 + 0.5t$$
$$t = 100 \text{ s} \qquad \textbf{Ans}$$

16-25. The disk starts from rest and is given an angular acceleration $\alpha = (10\theta^{1/3})$ rad/s^2, where θ is in radians. Determine the magnitudes of the normal and tangential components of acceleration of a point P on the rim of the disk when $t = 4$ s.



$\alpha = 10\theta^{\frac{1}{3}}$

$\omega\, d\omega = \alpha\, d\theta$

$\int_0^{\omega} \omega\, d\omega = \int_0^{\theta} 10\theta^{\frac{1}{3}}\, d\theta$

$\frac{1}{2}\omega^2 = 10\left(\frac{3}{4}\theta^{\frac{4}{3}}\right) = 7.5\theta^{\frac{4}{3}}$

$\omega = \frac{d\theta}{dt} = \sqrt{15}\theta^{\frac{2}{3}}$

$\int_0^{\theta} \theta^{-\frac{2}{3}}\, d\theta = \int_0^{t} \sqrt{15}\, dt$

$3\theta^{\frac{1}{3}} = \sqrt{15}\, t$

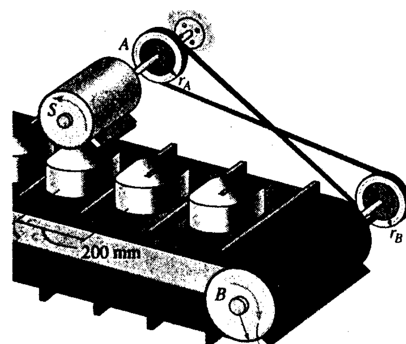
$\theta = 2.152t^3|_{t=4} = 137.71$

$\omega = \frac{d\theta}{dt} = 6.455t^2\Big|_{t=4} = 103.28$

$(a_P)_n = \omega^2 r = (103.28)^2(0.4) = 4267$ m/s^2 **Ans**

$(a_P)_t = \alpha r = \left(10(137.71)^{\frac{1}{3}}\right)(0.4) = 20.7$ m/s^2 **Ans**

16-31. A stamp S, located on the revolving drum, is used to label canisters. If the canisters are centered 200 mm apart on the conveyor, determine the radius r_A of the driving wheel A and the radius r_B of the conveyor belt drum so that for each revolution of the stamp it marks the top of a canister. How many canisters are marked per minute if the drum at B is rotating at $\omega_B = 0.2$ rad/s? Note that the driving belt is twisted as it passes between the wheels.



$l = 2\pi(r_A)$

$r_A = \dfrac{200}{2\pi} = 31.8$ mm **Ans**

For the drum at B :

$l = 2\pi(r_B)$

$r_B = \dfrac{200}{2\pi} = 31.8$ mm **Ans**

In $t = 60$ s,

$\theta = \theta_0 + \omega_0 t$

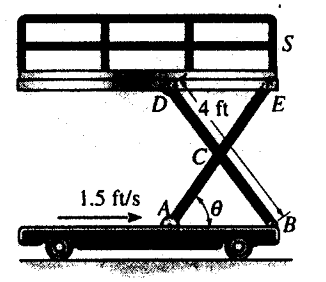
$\theta = 0 + 0.2(60) = 12$ rad

$l = \theta r_B = 12(31.8) = 382.0$ mm

Hence,

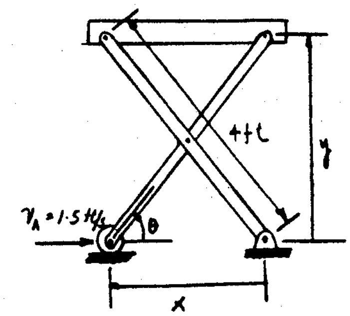
$n = \dfrac{382.0}{200} = 1.91$ canisters marked per minute **Ans**

16-34. The scaffold S is raised hydraulically by moving the roller at A toward the pin at B. If A is approaching B with a speed of 1.5 ft/s, determine the speed at which the platform is rising as a function of θ. The 4-ft links are pin-connected at their midpoint.



Position coordinate equation :

$$x = 4\cos\theta \qquad y = 4\sin\theta$$



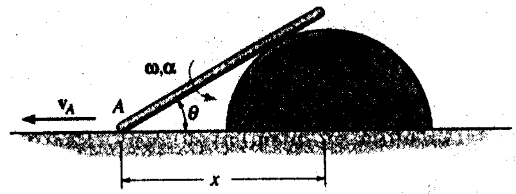
Time derivatives :

$$\dot{x} = -4\sin\theta\dot{\theta} \qquad \text{However,} \quad \dot{x} = -v_A = -1.5 \text{ ft/s}$$

$$-1.5 = -4\sin\theta\dot{\theta} \qquad \dot{\theta} = \frac{0.375}{\sin\theta}$$

$$\dot{y} = v_y = 4\cos\theta\dot{\theta} = 4\cos\theta\left(\frac{0.375}{\sin\theta}\right) = 1.5\cot\theta \qquad \textbf{Ans}$$

16-43. The end A of the bar is moving to the left with a constant velocity $\mathbf{v}_A$. Determine the angular velocity $\boldsymbol{\omega}$ and angular acceleration $\boldsymbol{\alpha}$ of the bar as a function of its position x.



Position Coordinate Equation : From the geometry.

$$x = \frac{r}{\sin\theta} \qquad [1]$$

Time Derivatives : Taking the time derivative of Eq. [1], we have

$$\frac{dx}{dt} = -\frac{r\cos\theta}{\sin^2\theta}\frac{d\theta}{dt} \qquad [2]$$

Since v_0 is directed toward positive x, then $\frac{dx}{dt} = v_A$. Also, $\frac{d\theta}{dt} = \omega$. From the geometry, $\sin\theta = \frac{r}{x}$ and $\cos\theta = \frac{\sqrt{x^2 - r^2}}{x}$. Substitute these values into Eq.[2], we have

$$v_A = -\left(\frac{r(\sqrt{x^2 - r^2}/x)}{(r/x)^2}\right)\omega$$

$$\omega = -\left(\frac{r}{x\sqrt{x^2 - r^2}}\right)v_A \qquad \textbf{Ans}$$

Taking the time derivative of Eq. [2], we have

$$\frac{d^2x}{dt^2} = \frac{r}{\sin^2\theta}\left[\left(\frac{1+\cos^2\theta}{\sin\theta}\right)\left(\frac{d\theta}{dt}\right)^2 - \cos\theta\frac{d^2\theta}{dt^2}\right] \qquad [3]$$

Here, $\frac{d^2x}{dt^2} = a = 0$ and $\frac{d^2\theta}{dt^2} = \alpha$. Substitute into Eq.[3], we have

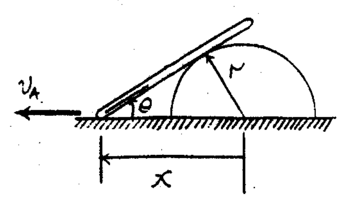
$$0 = \frac{r}{\sin^2\theta}\left[\left(\frac{1+\cos^2\theta}{\sin\theta}\right)\omega^2 - \alpha\cos\theta\right]$$

$$\alpha = \left(\frac{1+\cos^2\theta}{\sin\theta\cos\theta}\right)\omega^2 \qquad [4]$$

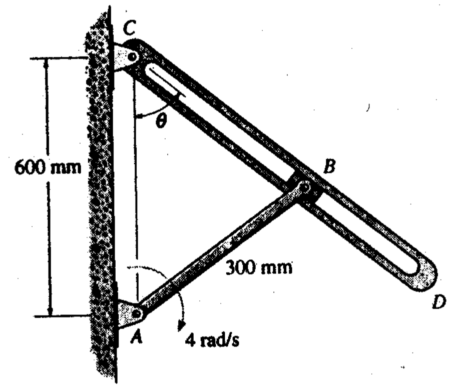
However, $\sin\theta = \frac{r}{x}$, $\cos\theta = \frac{\sqrt{x^2 - r^2}}{x}$ and $\omega = -\left(\frac{r}{x\sqrt{x^2 - r^2}}\right)v_A$.

Substitute these values into Eq.[4] yields

$$\alpha = \left[\frac{r(2x^2 - r^2)}{x^2(x^2 - r^2)^{3/2}}\right]v_A^2 \qquad \textbf{Ans}$$

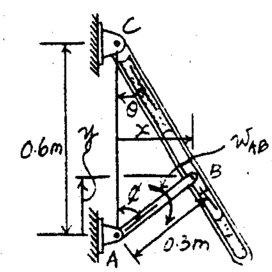


***16-48.** The crank AB is rotating with a constant angular velocity of 4 rad/s. Determine the angular velocity of the connecting rod CD at the instant $\theta = 30°$.



Position Coordinate Equation : From the geometry,

$$0.3\sin\phi = (0.6 - 0.3\cos\phi)\tan\theta \qquad [1]$$



Time Derivatives : Taking the time derivative of Eq. [1], we have

$$0.3\cos\phi\frac{d\phi}{dt} = 0.6\sec^2\theta\frac{d\theta}{dt} - 0.3\left(\cos\phi\sec^2\theta\frac{d\theta}{dt} - \tan\theta\sin\phi\frac{d\phi}{dt}\right)$$

$$\frac{d\theta}{dt} = \left[\frac{0.3(\cos\phi - \tan\theta\sin\phi)}{0.3\sec^2\theta(2-\cos\phi)}\right]\frac{d\phi}{dt} \qquad [2]$$

However, $\frac{d\theta}{dt} = \omega_{BC}$, $\frac{d\phi}{dt} = \omega_{AB} = 4$ rad/s. At the instant $\theta = 30°$, from Eq. [3], $\phi = 60.0°$. Substitute these values into Eq. [2] yields

$$\omega_{BC} = \left[\frac{0.3(\cos 60.0° - \tan 30°\sin 60.0°)}{0.3\sec^2 30°(2-\cos 60.0°)}\right](4) = 0 \qquad \textbf{Ans}$$