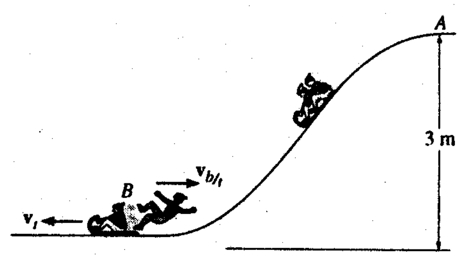
Homework #5 Solutions

15-34. A toboggan having a mass of 10 kg starts from rest at A and carries a girl and boy having a mass of 40 kg and 45 kg, respectively. When the toboggan reaches the bottom of the slope at B, the boy is pushed off from the back with a horizontal velocity of $v_{b/t} = 2$ m/s, measured relative to the toboggan. Determine the velocity of the toboggan afterwards. Neglect friction in the calculation.

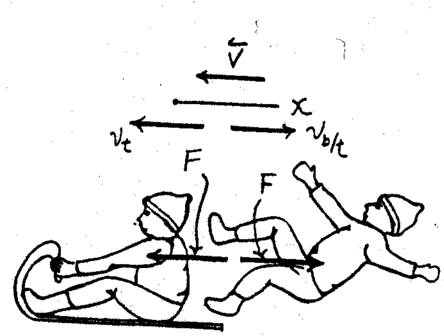


Conservation of Energy **:** The datum is set at the lowest point B. When the toboggan and its rider is at A, their position is 3 m *above* the datum and their gravitational potential energy is $(10+40+45)(9.81)(3) = 2795.85$ N · m. Applying Eq. 14 − 21, we have

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 2795.85 = \frac{1}{2}(10+40+45)\,v_B^2 + 0$$
$$v_B = 7.672 \text{ m/s}$$

Relative Velocity **:** The relative velocity of the falling boy with respect to the toboggan is $v_{b/t} = 2$ m/s. Thus, the velocity of the boy falling off the toboggan is

$$v_b = v_t + v_{b/t}$$
$$(\overset{+}{\leftarrow}) \qquad v_b = v_t - 2 \qquad [1]$$



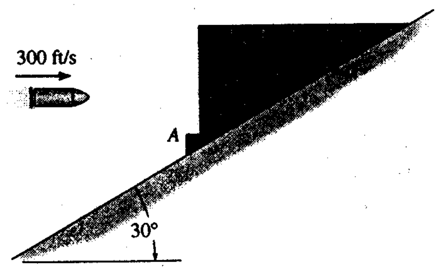
Conservation of Linear Momentum **:** If we consider the tobbogan and the riders as a system, then the impulsive force caused by the push is *internal* to the system. Therefore, it will cancel out. As the result, the linear momentum is conserved along the x axis.

$$m_T v_B = m_b v_b + (m_t + m_g)\,v_t$$
$$(\overset{+}{\leftarrow}) \qquad (10+40+45)(7.672) = 45v_b + (10+40)\,v_t \qquad [2]$$

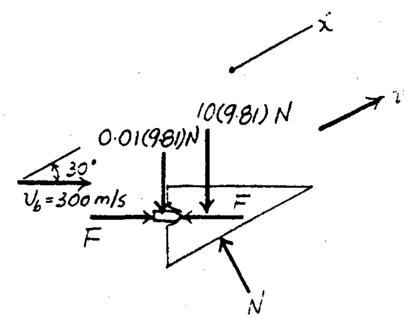
Solving Eqs. [1] and [2] yields

$$v_t = 8.62 \text{ m/s} \qquad \textbf{Ans}$$
$$v_b = 6.619 \text{ m/s}$$

15-47. The 10-kg block is held at rest on the smooth inclined plane by the stop block at A. If the 10-g bullet is traveling at 300 m/s when it becomes embedded in the 10-kg block, determine the distance the block will slide up along the plane before momentarily stopping.



Conservation of Linear Momentum : If we consider the block and the bullet as a system, then from the FBD, the *impulsive* force F caused by the impact is *internal* to the system. Therefore, it will cancel out. Also, the weight of the bullet and the block are *nonimpulsive forces*. As the result, linear momentum is conserved along the x' axis.



$$m_b(v_b)_{x'} = (m_b + m_B)v_x$$

$$(+\ )\qquad 0.01(300\cos 30°) = (0.01 + 10)\,v$$

$$v = 0.2595 \text{ m/s}$$

Conservation of Energy : The datum is set at the block's initial position. When the block and the embedded bullet is at their highest point, they are h *above* the datum. Their gravitational potential energy is $(10+0.01)(9.81)h = 98.1981h$. Applying Eq. 14 − 21, we have

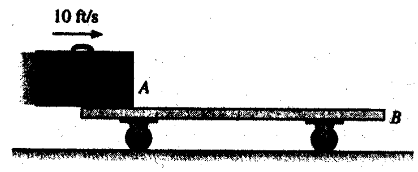
$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(10+0.01)\left(0.2595^2\right) = 0 + 98.1981h$$

$$h = 0.003433 \text{ m} = 3.43 \text{ mm}$$

$$d = 3.43 / \sin 30° = 6.87 \text{ mm} \qquad \textbf{Ans}$$

15-50. The 20-lb cart B is supported on rollers of negligible size. If a 10-lb suitcase A is thrown horizontally on it at 10 ft/s, determine the time t and the distance B moves before A stops relative to B. The coefficient of kinetic friction between A and B is $\mu_k = 0.4$.



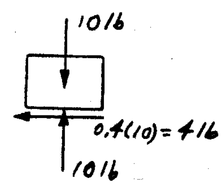
System :

$(\overset{+}{\rightarrow})\quad \Sigma m_1 v_1 = \Sigma m_2 v_2$

$$\left(\frac{10}{32.2}\right)(10) + 0 = \left(\frac{10+20}{32.2}\right)v$$

$v = 3.33$ ft/s

For A :



$$mv_1 + \Sigma\int F\,dt = mv_2$$

$$\left(\frac{10}{32.2}\right)(10) - 4t = \left(\frac{10}{32.2}\right)(3.33)$$

$t = 0.5176 = 0.518$ s **Ans**

For B :

$(\overset{+}{\rightarrow})\quad v = v_0 + a_c t$

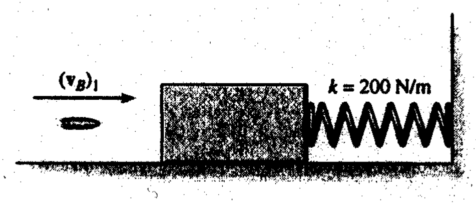
$3.33 = 0 + a_c\,(0.5176)$

$a_c = 6.440$ ft/s^2

$(\overset{+}{\rightarrow})\quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$s = 0 + 0 + \frac{1}{2}(6.440)(0.5176)^2 = 0.863$ ft **Ans**

R1-45. The 20-g bullet is fired horizontally at $(v_B)_1 =$ 1200 m/s into the 300-g block which rests on the smooth surface. Determine the distance the block moves to the right before momentarily coming to rest. The spring has a stiffness $k = 200$ N/m and is originally unstretched.



Impact

$\Sigma m v_1 = \Sigma m v_2$

$0.02(1200) + 0 = 0.320(v_2)$

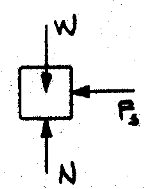
$v_2 = 75$ m/s

After collision;

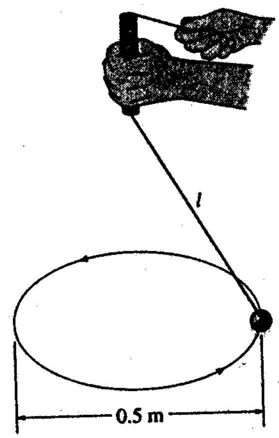
$T_1 + \Sigma U_{1-2} = T_2$

$\frac{1}{2}(0.320)(75)^2 - \frac{1}{2}(200)(x^2) = 0$

$x = 3$ m **Ans**



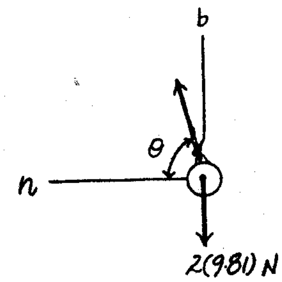
15-109. The 2-kg ball is rotating in a 0.5-m-diameter circular path with a constant speed. If the cord length is shortened from $l = 1$ m to $l' = 0.5$m, by pulling the cord through the tube, determine the new diameter of the path d'. Also, what is the tension in the cord in each case?



Equation of Motion : When the ball is travelling around the 0.5 m diameter circular path, $\cos\theta = \frac{0.25}{1} = 0.25$ and $\sin\theta = \frac{\sqrt{0.9375}}{1} = \sqrt{0.9375}$. Applying Eq. 13 − 8 , we have

$$\Sigma F_b = 0; \qquad T_1\left(\sqrt{0.9375}\right) - 2(9.81) = 0$$
$$T_1 = 20.26 \text{ N} = 20.3 \text{ N} \qquad \textbf{Ans}$$

$$\Sigma F_n = ma_n; \qquad 20.26(0.25) = 2\left(\frac{v_1^2}{0.25}\right)$$
$$v_1 = 0.7958 \text{ m/s}$$



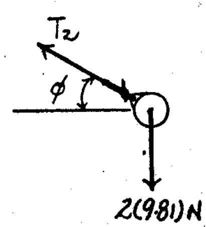
When the ball is travelling around d' the diameter circular path, $\cos\phi = \frac{d'/2}{0.5} = d'$ and $\sin\phi = \frac{\sqrt{0.25 - 0.25d'^2}}{0.5} = \sqrt{1 - d'^2}$. ApplyingEq. 13 − 8 , we have

$$\Sigma F_b = 0; \qquad T_2\left(\sqrt{1 - d'^2}\right) - 2(9.81) = 0 \qquad [1]$$

$$\Sigma F_n = ma_n; \qquad T_2(d') = 2\left(\frac{v_2^2}{d'/2}\right) \qquad [2]$$

Conservation of Angular Momentum : Since no force acts on the ball along the tangent of the circular path, the angular momentum is conserved about z axis. Applying Eq. 15 − 23, we have

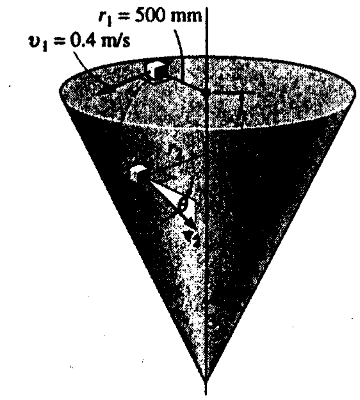
$$(\mathbf{H}_z)_1 = (\mathbf{H}_z)_2$$
$$r_1 m v_1 = r_2 m v_2$$
$$0.25(2)(0.7958) = \frac{d'}{2}(2)\,v_2 \qquad [3]$$



Solving Eqs. [1], [2] and [3] yields

$$d' = 0.41401 \text{ m} = 0.414 \text{ m} \qquad T_2 = 21.6 \text{ N} \qquad \textbf{Ans}$$
$$v_2 = 0.9610 \text{ m/s}$$

15-110. A small block having a mass of 0.1 kg is given a horizontal velocity $v_1 = 0.4$ m/s when $r_1 = 500$ mm. It slides along the smooth conical surface. When it descends to $h = 100$ mm, determine its speed and the angle of decent θ, that is, the angle measured from the horizontal to the tangent of the path.



Conservation of Angular Momentum : Since no force acts on the block along the tangent of the circular path the angular momentum is conserved about z axis. From the geometry, $r_2 = \left(\frac{0.5}{\tan 30^\circ} - 0.1\right)\tan 30^\circ = 0.4423$ m. Applying Eq. 15 – 23, we have

$$(H_z)_1 = (H_z)_2$$
$$r_1 m v_1 = r_2 m v'$$
$$0.5(0.1)(0.4) = 0.4423(0.1)\,v'$$
$$v' = 0.4522 \text{ m/s}$$

Conservation of Energy : Applying Eq. 14 – 21, we have

$$T_1 + V_1 = T_2 + V_2$$
$$\frac{1}{2}(0.1)\left(0.4^2\right) + 0.1(9.81)(0.1) = \frac{1}{2}(0.1)\,v_2^2 + 0$$
$$v_2 = 1.457 \text{ m/s} = 1.46 \text{ m/s} \qquad \textbf{Ans}$$

Thus, the angle of descent is given by

$$\theta = \cos^{-1}\frac{v'}{v_2} = \cos^{-1}\frac{0.4522}{1.457} = 71.9^\circ \qquad \textbf{Ans}$$