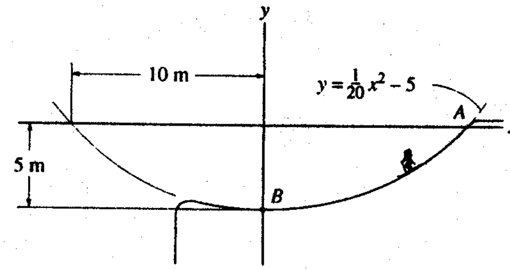
Homework #4 Solutions

13-79. The skier starts from rest at $A(10\text{ m}, 0)$ and descends the smooth slope, which may be approximated by a parabola. If she has a mass of 52 kg, determine the normal force she exerts on the ground at the instant she arrives at point B. Neglect the size of the skier. *Hint:* Use the result of Prob. 13–58.



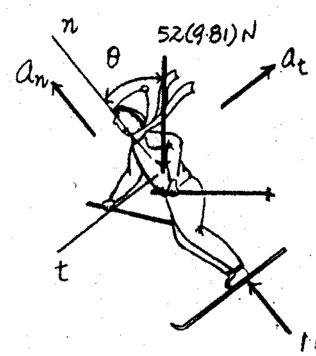
Geometry : Here, $\frac{dy}{dx} = \frac{1}{10}x$ and $\frac{d^2y}{dx^2} = \frac{1}{10}$. The slope angle θ at point B is given by

$$\tan\theta = \left.\frac{dy}{dx}\right|_{x=0\text{m}} = 0 \qquad \theta = 0^\circ$$

and the radius of curvature at point B is

$$\rho = \frac{[1+(dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1+\left(\frac{1}{10}x\right)^2\right]^{3/2}}{|1/10|}\right|_{x=0\text{m}} = 10.0\text{ m}$$

Equation of Motion :



$$\Sigma F_t = ma_t; \qquad 52(9.81)\sin\theta = -52a_t \qquad a_t = -9.81\sin\theta$$

$$\Sigma F_n = ma_n; \qquad N - 52(9.81)\cos\theta = m\left(\frac{v^2}{\rho}\right) \qquad [1]$$

Kinematics : The speed of the skier can be determined using $v\,dv = a_t\,ds$. Here, a_t must be in the direction of positive ds. Also, $ds = \sqrt{1+(dy/dx)^2}\,dx$ $= \sqrt{1+\frac{1}{100}x^2}\,dx$. Here, $\tan\theta = \frac{1}{10}x$. Then, $\sin\theta = \frac{x}{10\sqrt{1+\frac{1}{100}x^2}}$.

$$(+\ ) \quad \int_0^v v\,dv = -9.81\int_{10\text{m}}^{0}\left(\frac{x}{10\sqrt{1+\frac{1}{100}x^2}}\right)\left(\sqrt{1+\frac{1}{100}x^2}\,dx\right)$$

$$v^2 = 98.1\text{ m}^2/\text{s}^2$$

Also, one can obtained v^2 by using the result of Prob. 13 – 48.

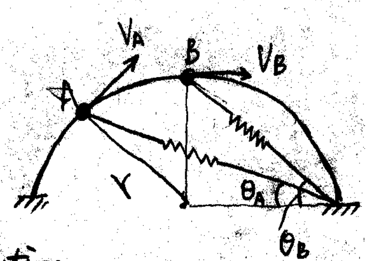
$$v^2 = 2gh = 2(9.81)(5) = 98.1\text{ m}^2/\text{s}^2$$

Substitute $v^2 = 98.1\text{ m}^2/\text{s}^2$, $\theta = 0^\circ$ and $\rho = 10.0$ m into Eq. [1] yields

$$N - 52(9.81)\cos 0^\circ = 52\left(\frac{98.1}{10.0}\right)$$

$$N = 1020.24\text{ N} = 1.02\text{ kN} \qquad \textbf{Ans}$$

13-82.



$m = 5\ kg$ $\quad v_A = 2\ m/s$

$k = 40\ N/m$ $\quad \theta_A = 30°$

$L_0 = 0.2\ m$ $\quad \theta_B = 45°$

$r = 1\ m$

Solution:

(1). Conservation of Mechanical Energy: $E_A = E_B$.

$$\left(\frac{1}{2}mv^2 + \frac{1}{2}k(L-L_0)^2\right)_A = \left(\frac{1}{2}mv^2 + \frac{1}{2}k(L-L_0)^2\right)_B$$

$$\frac{1}{2}mv_A^2 + \frac{1}{2}k(2r\cos\theta_A - L_0)^2 = \frac{1}{2}mv_B^2 + \frac{1}{2}k(2r\cos\theta_B - L_0)^2$$

$$v_B^2 = v_A^2 + \frac{k}{m}\left[(2r\cos\theta_A - L_0)^2 - (2r\cos\theta_B - L_0)^2\right]$$

$$v_B^2 = (2\ m/s)^2 + \frac{40\ N/m}{5\ kg}\left[(2(1m)\cos 30° - 0.2m)^2 - (2(1m)\cos 45° - 0.2\ m)^2\right]$$

$$v_B^2 = 10.98\ m^2/s^2$$

$$\Rightarrow \boxed{v_B = 3.31\ m/s}$$

(2). $v^2 = v_A^2 + \frac{k}{m}(2r\cos\theta_A - L_0)^2 - \frac{k}{m}(2r\cos\theta - L_0)^2$

$$2v\frac{dv}{dt} = -2\frac{k}{m}(2r\cos\theta - L_0)(-2r\sin\theta)\cdot\frac{d\theta}{dt}$$

$$2v\frac{dv}{dt} = 4\frac{k}{m}(2r\cos\theta - L_0)\sin\theta\cdot v \qquad \left(\text{since } v = r\frac{d\theta}{dt}\right)$$

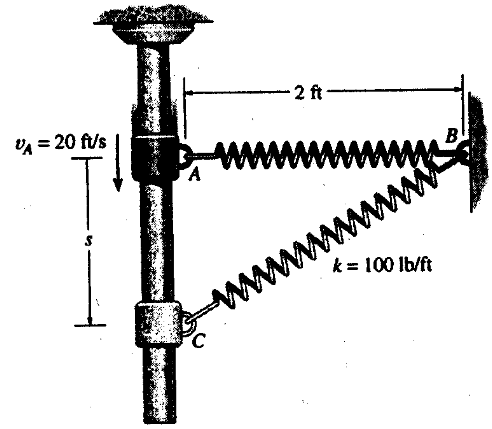
$$\Rightarrow \frac{dv}{dt} = \frac{2k\sin\theta}{m}(2r\cos\theta - L_0) = \frac{2(40\ N/m)\sin 45°}{5\ kg}(2(1m)\cos 45° - 0.2m)$$

$$\frac{dv}{dt} = 13.73\ m/s^2 = a_t$$

$$a_n = \frac{v^2}{r} = \frac{(3.31\ m/s)^2}{1m} = 10.98\ m/s^2 = a_n$$

$$\Rightarrow \boxed{a = (13.73\ \hat{u}_t + 10.98\ \hat{u}_n)\ m/s^2}$$

14-79. The 2-lb block is given an initial velocity of 20 ft/s when it is at A. If the spring has an unstretched length of 2 ft and a stiffness of $k = 100$ lb/ft, determine the velocity of the block when $s = 1$ ft.



Potential Energy : Datum is set along AB. The collar is 1 ft *below* the datum when it is at C. Thus, its gravitational potential energy at this point is $-2(1) = -2.00$ ft · lb. The initial and final elastic potential energy are $\frac{1}{2}(100)(2-2)^2 = 0$ and $\frac{1}{2}(100)\left(\sqrt{2^2+1^2}-2\right)^2 = 2.786$ ft · lb, respectively.

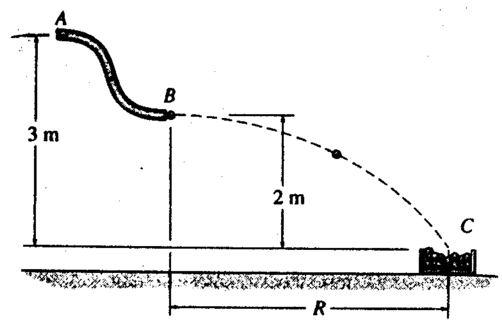
Conservation of Energy :

$$T_A + V_A = T_C + V_C$$

$$\frac{1}{2}\left(\frac{2}{32.2}\right)(20^2) + 0 = \frac{1}{2}\left(\frac{2}{32.2}\right)v_C^2 + 2.786 + (-2.00)$$

$$v_C = 19.4 \text{ ft/s}$$ **Ans**

14-83. Marbles having a mass of 5 g fall from rest at A through the glass tube and accumulate in the can at C. Determine the placement R of the can from the end of the tube and the speed at which the marbles fall into the can. Neglect the size of the can.



Potential Energy : Datum is set at point A. When the marble is at point B, its position is $(3-2) = 1$ m *below* the datum. Thus, its gravitational potential energy at this point is $0.005(9.81)(-1) = -0.04905$ N · m.

Conservation of Energy :

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 0 = \frac{1}{2}(0.005)v^2 + (-0.04905)$$

$$v = 4.429 \text{ m/s}$$

Kinematics : By considering the vertical motion, the vertical component of initial velocity is $(v_0)_y = 0$. When the marble travel from A to B, the initial and final vertical position are $(s_0)_y = 0$ and $s_y = -2$ m respectively.

$(+\uparrow)$
$$s_y = (s_0)_y + (v_0)_y t + \frac{1}{2}(a_c)_y t^2$$

$$-2 = 0 + 0 + \frac{1}{2}(-9.81)t^2$$

$$t = 0.6386 \text{ s}$$

$(+\uparrow)$
$$v_y = (v_0)_y + a_c t$$

$$v_y = 0 + (-9.81)(0.6386) = 6.264 \text{ m/s}$$

By considering the horizontal motion, the horizontal component of the velocity is $v_x = v = 4.429$ m/s. The traveling time is $t = 0.6386$ s.

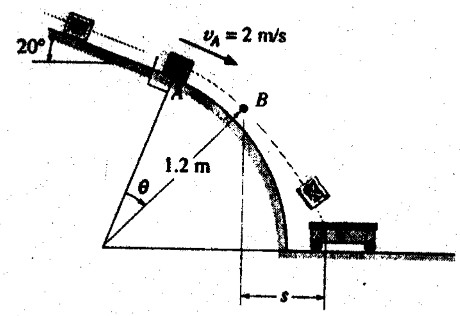
$(\overset{+}{\rightarrow})$
$$s_x = (s_0)_x + (v_0)_x t$$

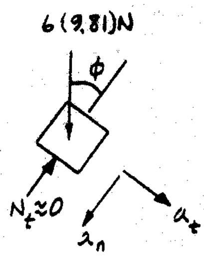
$$R = 0 + 4.429(0.6386) = 2.83 \text{ m}$$ **Ans**

The speed the marble hits the can is given by

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{4.429^2 + 6.264^2} = 7.67 \text{ m/s}$$ **Ans**

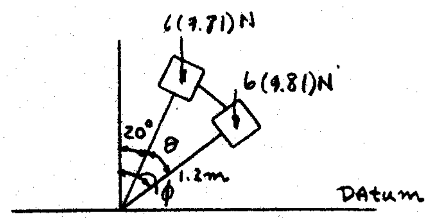
14-86. When the 6-kg box reaches point A it has a speed of $v_A = 2$ m/s. Determine the angle θ at which it leaves the smooth circular ramp and the distance s to where it falls into the cart. Neglect friction.





At point B :

$\swarrow^+ \Sigma F_n = ma_n$: $\quad 6(9.81)\cos\phi = 6\left(\frac{v_B^2}{1.2}\right) \quad (1)$



Datum at bottom of curve :

$T_A + V_A = T_B + V_B$

$\frac{1}{2}(6)(2)^2 + 6(9.81)(1.2\cos 20°) = \frac{1}{2}(6)(v_B)^2 + 6(9.81)(1.2\cos\phi)$

$13.062 = 0.5v_B^2 + 11.772\cos\phi \quad (2)$

Substitute Eq. (1) into Eq. (2), and solving for v_B,

$v_B = 2.951$ m/s

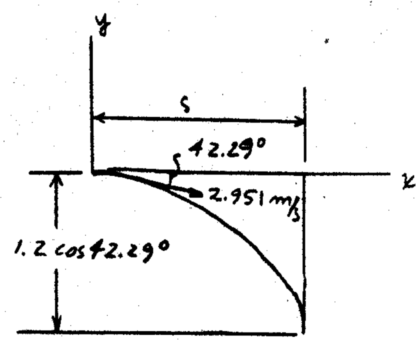
Thus, $\quad \phi = \cos^{-1}\left(\frac{(2.951)^2}{1.2(9.81)}\right) = 42.29°$

$\theta = \phi - 20° = 22.3°$ **Ans**

$(+\uparrow) \quad s = s_0 + v_0 t + \frac{1}{2}a_c t^2$

$-1.2\cos 42.29° = 0 - 2.951(\sin 42.29°)t + \frac{1}{2}(-9.81)t^2$

$4.905t^2 + 1.9857t - 0.8877 = 0$



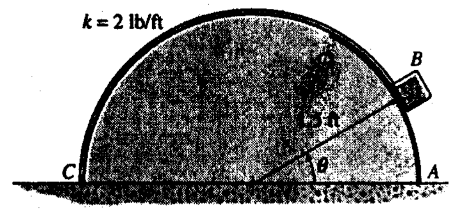
Solving for the positive root : $\quad t = 0.2687$ s

$(\overset{+}{\rightarrow}) \quad s = s_0 + v_0 t$

$s = 0 + (2.951\cos 42.29°)(0.2687)$

$s = 0.587$ m **Ans**

14-95. A 2-lb block rests on the smooth cylindrical surface at A. An elastic cord having a stiffness of $k = 2$ lb/ft is attached to the block at B and to the base of the cylinder at C. If the block is released from rest at A, determine the longest unstretched length of the cord so the block begins to leave the cylinder at the instant $\theta = 45°$. Neglect the size of the block.



Equation of Motion : It is required that $N = 0$. Applying Eq. 13 – 8 , we have

$$\Sigma F_n = ma_n; \qquad 2\cos 45° = \frac{2}{32.2}\left(\frac{v^2}{1.5}\right) \qquad v^2 = 34.15\ \text{m}^2/\text{s}^2$$

Potential Energy : Datum is set at the base of cylinder. When the block moves to a position $1.5\sin 45° = 1.061$ ft *above* the datum, its gravitational potential energy at this position is $2(1.061) = 2.121$ ft · lb. The initial and final elastic potential energy are $\frac{1}{2}(2)[\pi(1.5) - l]^2$ and $\frac{1}{2}(2)[0.75\pi(1.5) - l]^2$, respectively.

Conservation of Energy :

$$\Sigma T_1 + \Sigma V_1 = \Sigma T_2 + \Sigma V_2$$

$$0 + \frac{1}{2}(2)[\pi(1.5) - l]^2 = \frac{1}{2}\left(\frac{2}{32.2}\right)(34.15) + 2.121 + \frac{1}{2}(2)[0.75\pi(1.5) - l]^2$$

$$l = 2.77 \text{ ft} \qquad \textbf{Ans}$$