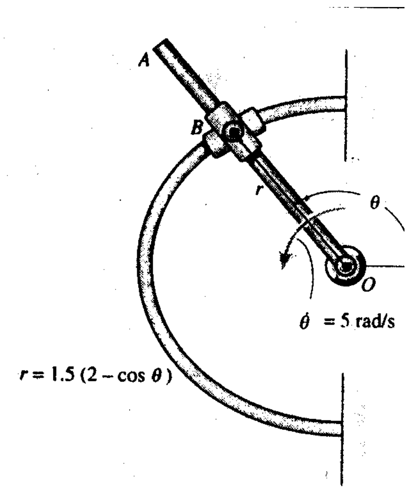
Homework #3 Solutions

13-89. Rod *OA* rotates counterclockwise with a constant angular velocity of $\dot{\theta} = 5$ rad/s. The double collar *B* is pin-connected together such that one collar slides over the rotating rod and the other slides over the *horizontal* curved rod, of which the shape is described by the equation $r = 1.5(2 - \cos\theta)$ ft. If both collars weigh 0.75 lb, determine the normal force which the curved rod exerts on one collar at the instant $\theta = 120°$. Neglect friction.

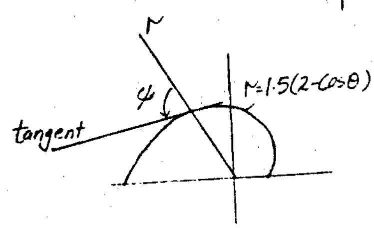


Kinematic : Here, $\dot{\theta} = 5$ rad/s and $\ddot{\theta} = 0$. Taking the required time derivatives at $\theta = 120°$, we have

$$r = 1.5(2 - \cos\theta)\big|_{\theta=120°} = 3.75 \text{ ft}$$
$$\dot{r} = 1.5\sin\theta\dot{\theta}\big|_{\theta=120°} = 6.495 \text{ ft/s}$$
$$\ddot{r} = 1.5\left(\sin\theta\ddot{\theta} + \cos\theta\dot{\theta}^2\right)\big|_{\theta=120°} = -18.75 \text{ ft/s}^2$$

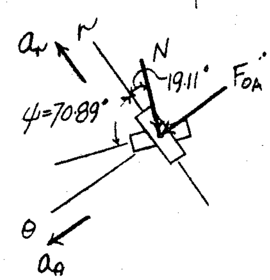
Applying Eqs. 12 − 29, we have

$$a_r = \ddot{r} - r\dot{\theta}^2 = -18.75 - 3.75\left(5^2\right) = -112.5 \text{ ft/s}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3.75(0) + 2(6.495)(5) = 64.952 \text{ ft/s}^2$$



Equation of Motion : The angle ψ must be obtained first.

$$\tan\psi = \frac{r}{dr/d\theta} = \frac{1.5(2-\cos\theta)}{1.5\sin\theta}\bigg|_{\theta=120°} = 2.8867 \qquad \psi = 70.89°$$

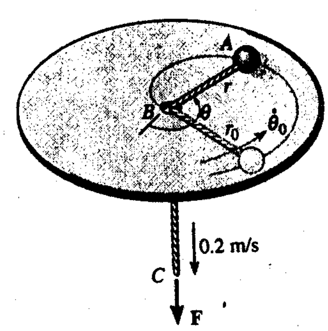


Applying Eq. 13 − 9, we have

$$\Sigma F_r = ma_r; \qquad -N\cos 19.11° = \frac{0.75}{32.2}(-112.5)$$
$$N = 2.773 \text{ lb} = 2.77 \text{ lb} \qquad \textbf{Ans}$$

$$\Sigma F_\theta = ma_\theta; \qquad F_{OA} + 2.773\sin 19.11° = \frac{0.75}{32.2}(64.952)$$
$$F_{OA} = 0.605 \text{ lb} \qquad \textbf{Ans}$$

13-101. The ball has a mass of 2 kg and a negligible size. It is originally traveling around the horizontal circular path of radius $r_0 = 0.5$ m such that the angular rate of rotation is $\dot{\theta}_0 = 1$ rad/s. If the attached cord ABC is drawn down through the hole at a constant speed of 0.2 m/s, determine the tension the cord exerts on the ball at the instant $r = 0.25$ m. Also, compute the angular velocity of the ball at this instant. Neglect the effects of friction between the ball and horizontal plane. *Hint:* First show that the equation of motion in the θ direction yields $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (1/r)(d(r^2\dot{\theta})/dt) = 0$. When integrated, $r^2\dot{\theta} = c$, where the constant c is determined from the problem data.



$\Sigma F_\theta = m a_\theta$: $\quad 0 = m\left[r\,\theta + 2\,r\,\theta\right] = m\left[\frac{1}{r}\frac{d}{dt}(r^2\theta)\right] = 0$

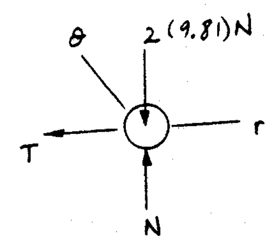
Thus,

$d(r^2\theta) = 0$

$r^2\theta = C$

$(0.5)^2(1) = C = (0.25)^2\,\theta$

$\theta = 4.00$ rad/s **Ans**



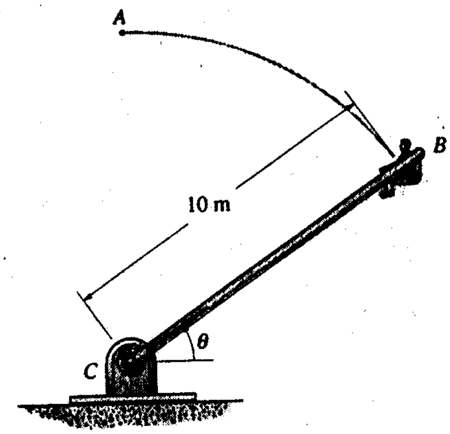
Since $r = -0.2$ m/s, $\quad r = 0$

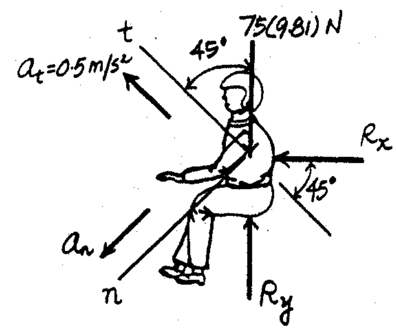
$a_r = r - r(\theta)^2 = 0 - 0.25(4.00)^2 = -4$ m/s^2

$\Sigma F_r = m\,a_r$; $\quad -T = 2(-4)$

$T = 8$ N **Ans**

13-59. The device shown is used to produce the experience of weightlessness in a passenger when he reaches point A, $\theta = 90°$, along the path. If the passenger has a mass of 75 kg, determine the minimum speed he should have when he reaches A so that he does not exert a normal reaction on the seat. The chair is pin-connected to the frame BC so that he is always seated in an upright position. During the motion his speed remains constant.



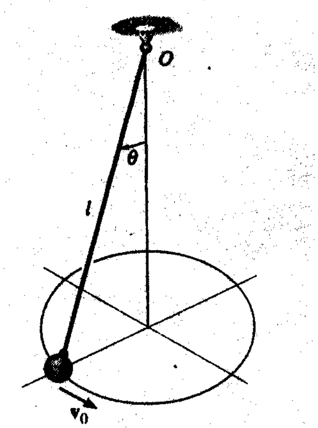


Equation of Motion : If the man is about to fly off from the seat, the normal reaction $N = 0$. Applying Eq. 13 – 8, we have

$$\Sigma F_n = ma_n; \qquad 75(9.81) = 75\left(\frac{v^2}{10}\right)$$

$$v = 9.90 \text{ m/s} \qquad \textbf{Ans}$$

13-71. The ball has a mass m and is attached to the cord of length l. The cord is tied at the top to a swivel and the ball is given a velocity $\mathbf{v}_0$. Show that the angle θ which the cord makes with the vertical as the ball travels around the circular path must satisfy the equation $\tan\theta\sin\theta = v_0^2/gl$. Neglect air resistance and the size of the ball.



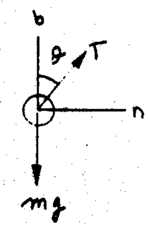
$\xrightarrow{+} \Sigma F_n = ma_n;\qquad T\sin\theta = m\left(\frac{v_0^2}{r}\right)$

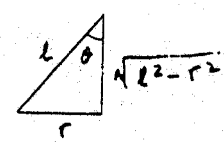
$+\uparrow \Sigma F_b = 0;\qquad T\cos\theta - mg = 0$

Since $r = l\sin\theta \qquad T = \frac{mv_0^2}{l\sin^2\theta}$

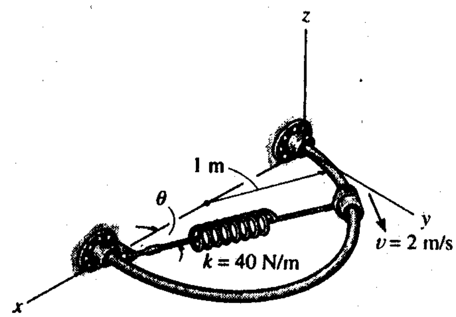
$$\left(\frac{mv_0^2}{l}\right)\left(\frac{\cos\theta}{\sin^2\theta}\right) = mg$$

$$\tan\theta\,\sin\theta = \frac{v_0^2}{gl} \qquad \textbf{Q.E.D.}$$

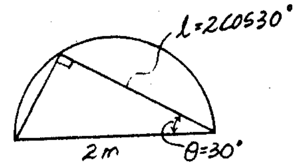




13-82. The collar has a mass of 5 kg and is confined to move along the smooth circular rod which lies in the horizontal plane. The attached spring has an unstretched length of 200 mm. If, at the instant $\theta = 30°$, the collar has a speed $v = 2\text{m/s}$, determine the magnitude of normal force of the rod on the collar and the collar's acceleration.



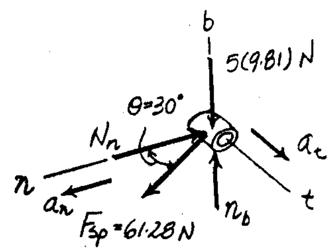
Equation of Motion : The spring force is given by $F_{sp} = k(l - l_0) = 40(2\cos 30° - 0.2) = 61.28\text{ N}$. The normal component of acceleration is $a_n = \frac{v^2}{\rho} = \frac{2^2}{1} = 4\text{ m/s}^2$. Applying Eq. 13-8, we have



$\Sigma F_b = 0;$ $\quad N_b - 5(9.81) = 0 \quad N_b = 49.05\text{ N}$

$\Sigma F_t = ma_t;$ $\quad 61.28\sin 30° = 5a_t \quad a_t = 6.128\text{ m/s}^2$

$\Sigma F_n = ma_n;$ $\quad 61.28\cos 30° - N_n = 5(4) \quad N_n = 33.07\text{ N}$



Thus, the magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{6.128^2 + 4^2} = 7.32\text{ m/s}^2 \qquad \textbf{Ans}$$

and the magnitude of normal force is

$$N = \sqrt{N_b^2 + N_n^2} = \sqrt{49.05^2 + 33.07^2} = 59.2\text{ N} \qquad \textbf{Ans}$$