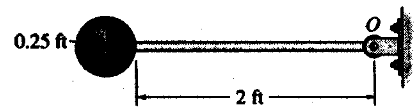
Homework #10 Solutions

17-54. The pendulum consists of a 20-lb sphere and a 5-lb slender rod. Determine the reaction at the pin O just after the pendulum is released from the position shown.



$$I_O = \left[\frac{2}{5}\left(\frac{20}{32.2}\right)(0.25)^2 + \left(\frac{20}{32.2}\right)(2.25)^2\right] + \left[\frac{1}{3}\left(\frac{5}{32.2}\right)(2)^2\right] = 3.367 \text{ slug}\cdot\text{ft}^2$$

$\xrightarrow{+} \Sigma F_x = m(a_G)_x; \quad O_x = 0$

$+\downarrow \Sigma F_y = m(a_G)_y; \quad 20 + 5 - O_y = \left(\frac{20}{32.2}\right)a_b + \left(\frac{5}{32.2}\right)a_R$

$+\Sigma M_O = I_O\alpha; \quad (20)(2.25) + (5)(1) = 3.367\alpha$

$a_b = (2.25)\alpha$

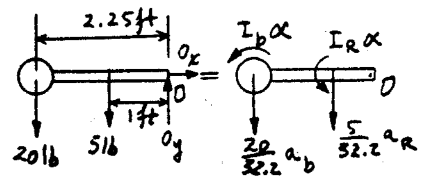
$a_R = (1)\alpha$

Solving,

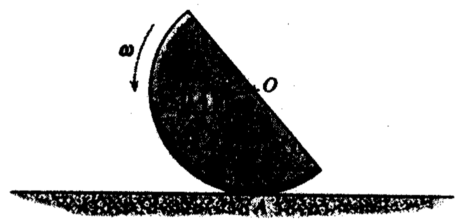
$O_x = 0$ **Ans**

$O_y = 1.94$ lb **Ans**

$\alpha = 14.9 \text{ rad/s}^2$ **Ans**



17-89. The semicircular disk having a mass of 10 kg is rotating at ω = 4 rad/s at the instant θ = 60°. If the coefficient of static friction at A is μ_s= 0.5, determine if the disk slips at this instant.

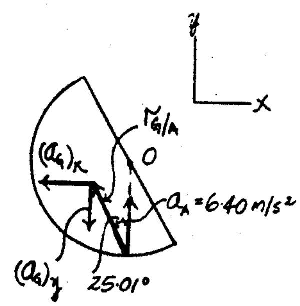


Equation of Motion : The mass moment of inertia of the semicircular disk about its center of mass is given by $I_G = \frac{1}{2}(10)(0.4^2) - 10(0.1698^2) = 0.5118\ \text{kg}\cdot\text{m}^2$. From the geometry, $r_{G/A} = \sqrt{0.1698^2 + 0.4^2 - 2(0.1698)(0.4)\cos 60^\circ} = 0.3477$ m. Also, using law of sines, $\frac{\sin\theta}{0.1698} = \frac{\sin 60^\circ}{0.3477}$, $\theta = 25.01°$. Applying Eq. 17 − 16, we have

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \quad 10(9.81)(0.1698\sin 60^\circ) = 0.5118\alpha + 10(a_G)_x \cos 25.01^\circ(0.3477) + 10(a_G)_y \sin 25.01^\circ(0.3477) \quad [1]$$

$$\overset{+}{\leftarrow} \Sigma F_x = m(a_G)_x; \quad F_f = 10(a_G)_x \quad [2]$$

$$+\uparrow F_y = m(a_G)_y; \quad N - 10(9.81) = -10(a_G)_y \quad [3]$$



Kinematics : Assume that the semicircular disk does not slip at A, then $(a_A)_x = 0$. Here, $\mathbf{r}_{G/A} = \{-0.3477\sin 25.01^\circ\mathbf{i} + 0.3477\cos 25.01^\circ\mathbf{j}\}$ m $= \{-0.1470\mathbf{i} + 0.3151\mathbf{j}\}$ m. Applying Eq. 16 − 18, we have

$$\mathbf{a}_G = \mathbf{a}_A + \alpha \times \mathbf{r}_{G/A} - \omega^2 \mathbf{r}_{G/A}$$
$$-(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = 6.40\mathbf{j} + \alpha\mathbf{k} \times (-0.1470\mathbf{i} + 0.3151\mathbf{j}) - 4^2(-0.1470\mathbf{i} + 0.3151\mathbf{j})$$
$$-(a_G)_x\mathbf{i} - (a_G)_y\mathbf{j} = (2.3523 - 0.3151\alpha)\mathbf{i} + (1.3581 - 0.1470\alpha)\mathbf{j}$$

Equating **i** and **j** components, we have

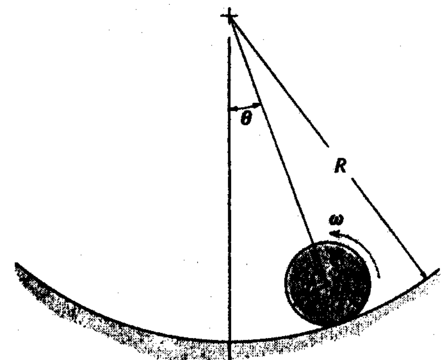
$$(a_G)_x = 0.3151\alpha - 2.3523 \quad [4]$$
$$(a_G)_y = 0.1470\alpha - 1.3581 \quad [5]$$

Solving Eqs. [1], [2], [3], [4] and [5] yields

$$\alpha = 13.85\ \text{rad/s}^2 \quad (a_G)_x = 2.012\ \text{m/s}^2 \quad (a_G)_y = 0.6779\ \text{m/s}^2$$
$$F_f = 20.12\ \text{N} \quad N = 91.32\ \text{N}$$

Since $F_f < (F_f)_{max} = \mu_s N = 0.5(91.32) = 45.66$ N, then the semicircular disk **does not slip.** **Ans**

17-98. The disk of mass m and radius r rolls without slipping on the circular path. Determine the normal forces which the path exerts on the disk and the disk's angular acceleration if at the instant shown the disk has an angular velocity of ω.



Equation of Motion : The mass moment of inertia of the disk about its center of mass is given by $I_G = \frac{1}{2}mr^2$. Applying Eq. 17 – 16, we have

$$\circlearrowleft + \Sigma M_A = \Sigma(M_k)_A; \qquad mg\sin\theta(r) = \left(\frac{1}{2}mr^2\right)\alpha + m(a_G)_t(r) \qquad [1]$$

$$\Sigma F_n = m(a_G)_n; \qquad N - mg\cos\theta = m(a_G)_n \qquad [2]$$

Kinematics : Since the semicircular disk does not slip at A, then $v_G = \omega r$ and $(a_G)_t = \alpha r$. Substitute $(a_G)_t = \alpha r$ into Eq. [1] yields

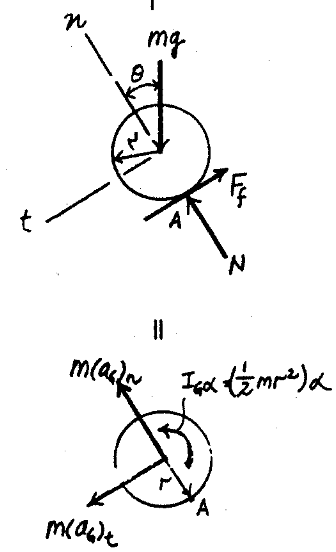
$$mg\sin\theta(r) = \left(\frac{1}{2}mr^2\right)\alpha + m(\alpha r)(r)$$

$$\alpha = \frac{2g}{3r}\sin\theta \qquad \textbf{Ans}$$

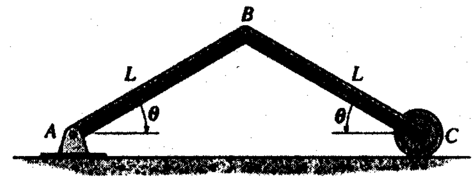
Also, the center of the mass for the disk moves around a circular path having a radius of $\rho = R - r$. Thus, $(a_G)_n = \frac{v_G^2}{\rho} = \frac{\omega^2 r^2}{R - r}$. Substitute into Eq. [2] yields

$$N - mg\cos\theta = m\left(\frac{\omega^2 r^2}{R - r}\right)$$

$$N = m\left(\frac{\omega^2 r^2}{R - r} + g\cos\theta\right) \qquad \textbf{Ans}$$



18-45. The two bars are released from rest at the position θ. Determine their angular velocities at the instant they become horizontal. Neglect the mass of the roller at C. Each bar has a mass m and length L.

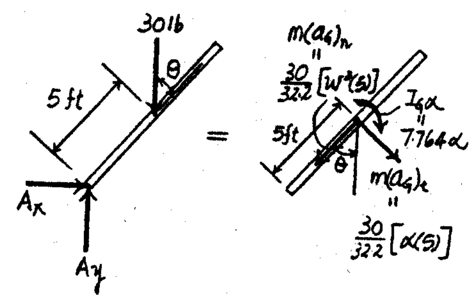


Potential Energy : Datum is set at point A. When links AB and BC is at their initial position, their center of gravity is located $\frac{L}{2}\sin\theta$ *above* the datum. Their gravitational potential energy at this position is $mg\left(\frac{L}{2}\sin\theta\right)$. Thus, the initial and final potential energies are

$$V_1 = 2\left(\frac{mgL}{2}\sin\theta\right) = mgL\sin\theta \qquad V_2 = 0$$

Kinetic Energy : When links AB and BC are in the horizontal position, then $v_B = \omega_{AB}L$ which is directed vertically downward since link AB is rotating about fixed point A. Link BC is subjected to general plane motion and its instantaneous center of zero velocity is located at point C. Thus, $v_B = \omega_{BC}r_{B/IC}$ or $\omega_{AB}L = \omega_{BC}L$, hence $\omega_{AB} = \omega_{BC} = \omega$. The mass moment inertia for link AB and BC about point A and C is $(I_{AB})_A = (I_{BC})_C = \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{3}mL^2$. Since links AB and CD are at rest initially, the initial kinetic energy is $T_1 = 0$. The final kinetic energy is given by

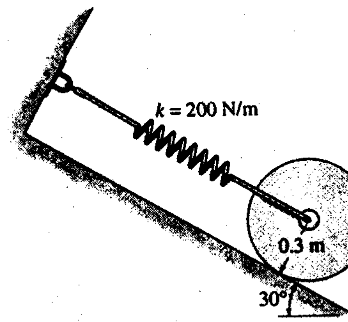
$$T_2 = \frac{1}{2}(I_{AB})_A\,\omega_{AB}^2 + \frac{1}{2}(I_{BC})_C\,\omega_{BC}^2$$
$$= \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2 + \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$
$$= \frac{1}{3}mL^2\omega^2$$



Conservation of Energy : Applying Eq. 18 – 18, we have

$$T_1 + V_1 = T_2 + V_2$$
$$0 + mgL\sin\theta = \frac{1}{3}mL^2\omega^2 + 0$$
$$\omega_{AB} = \omega_{BC} = \omega = \sqrt{\frac{3g}{L}\sin\theta} \qquad \textbf{Ans}$$

***18-48.** At the instant the spring becomes undeformed, the center of the 40-kg disk has a speed of 4 m/s. From this point determine the distance d the disk moves down the plane before momentarily stopping. The disk rolls without slipping.



Datum at lowest point.

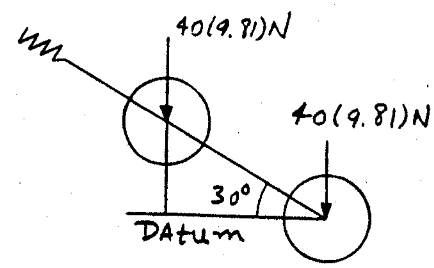
$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left[\frac{1}{2}(40)(0.3)^2\right]\left(\frac{4}{0.3}\right)^2 + \frac{1}{2}(40)(4)^2 + 40(9.81)d\sin 30° = 0 + \frac{1}{2}(200)d^2$$

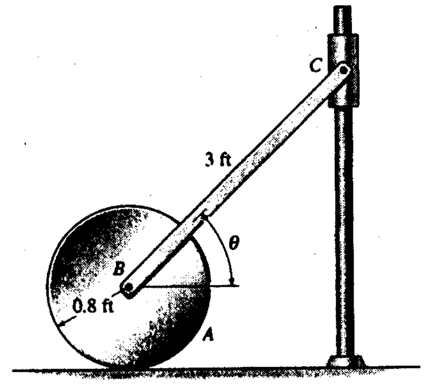
$$100d^2 - 196.2d - 480 = 0$$

Solving for the positive root

$d = 3.38$ m **Ans**

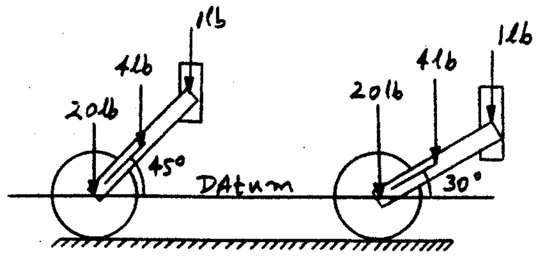


18-53. The system consists of a 20-lb disk A, 4-lb slender rod BC, and a 1-lb smooth collar C. If the disk rolls without slipping, determine the velocity of the collar at the instant $\theta = 30°$. The system is released from rest when $\theta = 45°$.



$v_B = 0.8\omega_D$

$$\omega_{BC} = \frac{v_B}{1.5} = \frac{v_C}{2.598} = \frac{v_G}{1.5}$$



Thus,

$v_B = v_G = 1.5\omega_{BC}$

$v_C = 2.598\omega_{BC}$

$\omega_D = 1.875\omega_{BC}$

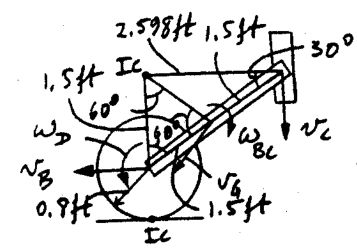
$T_1 + V_1 = T_2 + V_2$

$$0 + 4(1.5\sin45°) + 1(3\sin45°) = \frac{1}{2}\left[\frac{1}{2}\left(\frac{20}{32.2}\right)(0.8)^2\right](1.875\omega_{BC})^2 + \frac{1}{2}\left(\frac{20}{32.2}\right)(1.5\omega_{BC})^2$$

$$+\frac{1}{2}\left[\frac{1}{12}\left(\frac{4}{32.2}\right)(3)^2\right]\omega_{BC}^2 + \frac{1}{2}\left(\frac{4}{32.2}\right)(1.5\omega_{BC})^2$$

$$+\frac{1}{2}\left(\frac{1}{32.2}\right)(2.598\omega_{BC})^2 + 4(1.5\sin30°) + 1(3\sin30°)$$

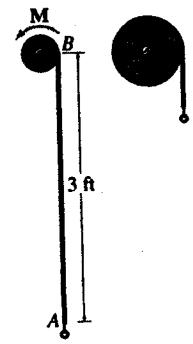
$$\omega_{BC} = 1.180 \text{ rad/s}$$



Thus

$v_C = 2.598(1.180) = 3.07$ ft/s **Ans**

18-59. The uniform window shade AB has a total weight of 0.4 lb. When it is released, it winds up around the spring-loaded core O. Motion is caused by a spring within the core, which is coiled so that it exerts a torque $M = 0.3(10^{-3})\theta$ lb·ft, where θ is in radians, on the core. If the shade is released from rest, determine the angular velocity of the core at the instant the shade is completely rolled up, i.e., after 12 revolutions. When this occurs, the spring becomes uncoiled and the radius of gyration of the shade about the axle at O is $k_O = 0.9$ in. *Note:* The elastic potential energy of the torsional spring is $V_e = \frac{1}{2}k\theta^2$, where $M = k\theta$ and $k = 0.3(10^{-3})$ lb·ft/rad.



$$T_1 + V_1 = T_2 + V_2$$

$$0-(0.4)(1.5)+\frac{1}{2}(0.3)(10^{-3})(24\pi)^2 = \frac{1}{2}\left(\frac{0.4}{32.2}\right)\left(\frac{0.9}{12}\right)^2\omega^2$$

$\omega = 85.1$ rad/s **Ans**