24-311 NUMERICAL METHODS Fall 03

Carnegie Mellon University

PROBLEM SET 9

Issued: 10/24/2003
Due: 10/31/2003 3:00PM @ HH B127
Weight: 4% of total grade

PS9-1 Statistics

Given the data

15	6	18	21	26	28	32
39	22	28	24	27	27	33
2	12	11	34	39	31	38
45	36	41	37	43	38	26

determine (a) the mean, (b) the standard deviation, (c) the variance, (d) the coefficient of variation.

PS9-2 Least-Square Regression

Suppose you want to fit a quadratic curve (parabola) to the following data points using least square regression.

x	0	1	2	3	4	5
y	0	2	2	1	-1	-2

(1) What is the function S_r to be minimized to find the least square regression curve?

(2) What are the three conditions to find the coefficients of the quadratic curve of the least square regression? Write the matrix equation of the three conditions.

PS9-3 Two Dimensional Quadratic Regression

Read and understand section 17.3 of the textbook. The mathematical background of this programming assignment is very similar to the one in section 17.3.

The goal of two dimensional quadratic regression is to find the "best" values of the coefficients of a quadratic function y of x_1 and x_2

$$y = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_1 x_2 + a_4 x_1^2 + a_5 x_2^2 .$$

The coefficients $a_0, a_1, a_2, a_3, a_4,$ and a_5 can be determined such that they define a function y that minimizes the sum of the squares of the errors, or residuals,

$$S_r = \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_1 - a_2 x_2 - a_3 x_1 x_2 - a_4 x_1^2 - a_5 x_2^2 \right)^2$$

Differentiation of S_r with respect to each of the unknown coefficients, $a_0, a_1, a_2, a_3, a_4,$ and a_5, gives six linear algebraic equations, or a 6 x 6 matrix equation, [M]{a}={b}, similar to equation (17.22) on p.461 of the textbook.

(1) Derive the matrix equation [M]{a}={b} for two-dimensional quadrilateral regression. Show all the derivation process.

(2) Given the following data points write a program for finding coefficients $a_0, a_1, a_2, a_3, a_4,$ and a_5 of the quadratic regression function.

	x2 = 0.0	x2 = 0.2	x2 = 0.4	x2 = 0.6	x2 = 0.8	x2 = 1.0
x1 = 0.0	0.7	0.857	0.81	0.849	0.711	0.456
x1 = 0.2	0.537	0.55	0.649	0.571	0.376	0.247
x1 = 0.4	0.33	0.489	0.471	0.336	0.267	-0.039
x1 = 0.6	0.369	0.411	0.386	0.327	0.081	-0.181
x1 = 0.8	0.391	0.379	0.427	0.241	0.039	-0.172
x1 = 1.0	0.459	0.567	0.441	0.299	0.148	-0.243

To show that your program is properly implemented, your program has to generate: (1) a text file, `output.txt`, that contains the 6 x 6 matrix [M], and values of coefficients $a_0, a_1, a_2, a_3, a_4,$ and a_5; and (2) a VRML file, `output.wrl`, that illustrates the data points and the quadratic regression surface.

Your VRML file should look something like Figure 1. To minimize your work to generate the VRML output the following subroutines and a sample code are provided.

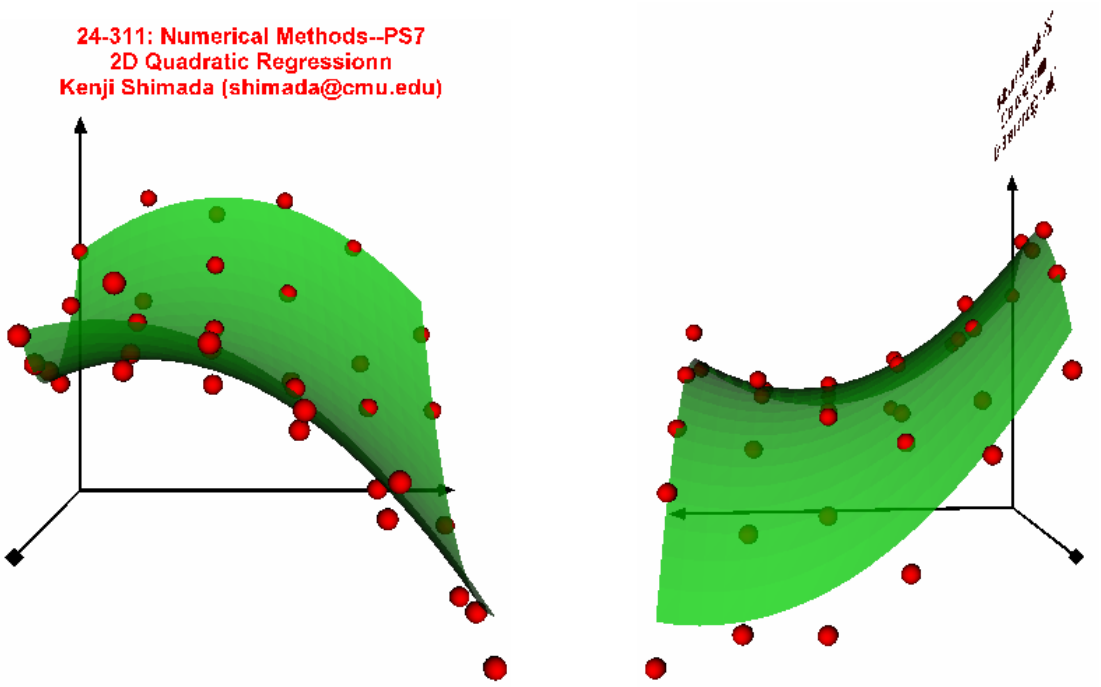
```
void vrml_reg (ofstream &outFile, char* my_name, double a[])
// Draw in the output vrml file, axes, data points, and surface
//  outFile: pointer to the output vrml file
//  my_name: my name to be shown in the heading
//  a[]:     coefficients of qadratic surface
```

In your hand-in directory on AFS, make a new directory called ps9 (in lower case) and hand in the following in the directory. Don't copy object files (*.obj, *.o).

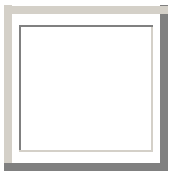
- Source code files and header file
- Executable file
- Text file name `output.txt`
- Output VRML file named `output.wrl`.

Also hand in a printout of the following:

- Source code files and header files
- Text file, `ourput.txt`
- Two images of the VRML files that look like Figure 1.



PS9



The first letter of your LAST name

First Name _____ Last Name _____

PS9-1 (20 pts)	PS9-2 (20 pts)	PS9-3a (30 pts) For handing in all the files and printouts (code and wrl files)	PS9-3b (30 pts) For the correct method and the results	Total (100 pts)

24-311 NUMERICAL METHODS Fall 03

Carnegie Mellon University

PROBLEM SET 9

Issued: 10/24/2003
Due: 10/31/2003 3:00PM @ HH B127
Weight: 4% of total grade