24-311 NUMERICAL METHODS Fall 03

Carnegie Mellon University

PROBLEM SET 7

Issued: 10/10/2003
Due: 10/17/2003 4:00PM @ HH B127
Weight: 3% of total grade

PS7-1 Directional Derivative

Find the directional derivative of $f(x,y) = 2x^2 + y^2$ at $x = 2$ and $y = 2$ in the direction of $\mathbf{h} = 3\mathbf{i} + 2\mathbf{j}$. (Read the textbook pp. 360-363 for the definition of the directional derivative.)

PS7-2 Gradient Vector and Hessian Matrix

Find the gradient vector and Hessian matrix for $f(x,y) = 2xy^2 + 3e^{xy}$

PS7-3 Necessary Conditions for Min/Max

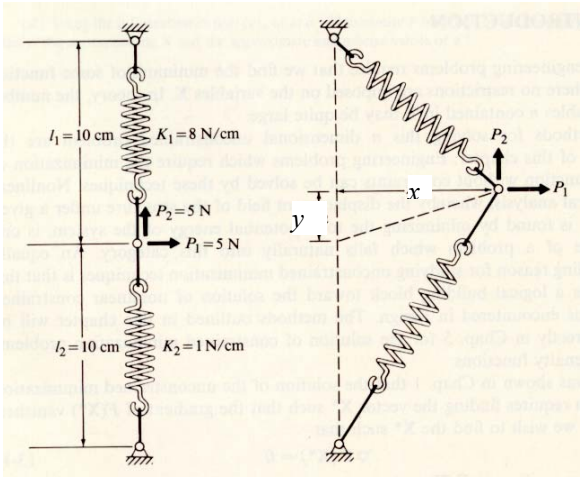
Given

$$f(x,y) = 2xy + 1.5y - 1.25x^2 - 2y^2,$$

construct and solve a system of linear algebraic equations, or a matrix equation, that maximizes $f(x,y)$. Note that this is done by setting the partial derivatives of the function with respect to x and y to zero.

PS7-4 2D Unconstrained Optimization--Equilibrium Position of a Spring System

Consider the simple two-spring system shown in the following figure. The springs are assumed to be linearly elastic and the loads $p_1 = 5\,\text{N}$ and $p_2 = 5\,\text{N}$ are constant. This is a geometrically nonlinear problem because the resistance to the load is a function of the deformed position. The original length of the two springs are $l_1 = 10\,\text{cm}$ and $l_2 = 10\,\text{cm}$, and two spring constants are $k_1 = 8\,\text{N/cm}$ and $k_2 = 1\,\text{N/cm}$.



(1) Find the total energy of the system $E(x, y)$ as a function of x and y.

(2) Use Mathcad to draw the surface plot of the function, $E(x, y)$. Use the "color" shading option. Your plot should look similar to Figure 2.

(3) Use Mathcad to draw the contour plot of the function, $E(x, y)$. Draw at least 40 contours. Your plot should look similar to Figure 3.

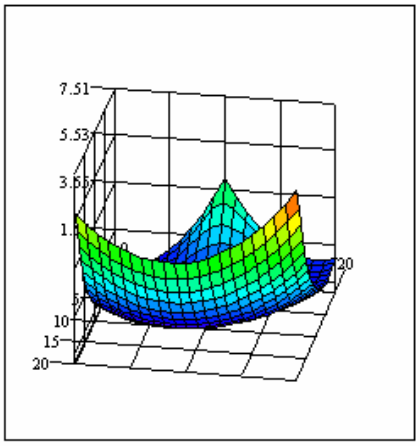


Figure 2. Surface plot

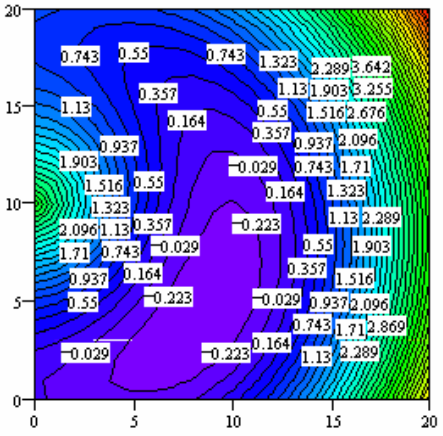
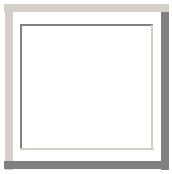


Figure 3. Contour plot

PS7



The first letter of your LAST name

First Name _____ Last Name _____

PS7-1 (20 pts)	PS7-2 (20 pts)	PS7-3 (20 pts)	PS7-4 (40 pts)	Total (100 pts)

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