24-311 NUMERICAL METHODS Fall 02

Carnegie Mellon University

PROBLEM SET 2

Issued: 9/4/02
Due: 9/11/02 (Please hand in your solutions at the beginning of class.)
Weight: 4% of total grade

PS2-1 C++/Java Programming—Forward Finite Divided Difference

This is essentially the same problem as PS1-3, but you will solve it by writing a C++ or JAVA code.

A storage tank contains a liquid at depth y, where $y = 0$ when the tank is half full. Liquid is withdrawn at a constant flow rate Q to meet demands. The contents are re-supplied at a sinusoidal rate $3Q\sin^2(t)$ (see Figure 1). The cross-sectional area of the tank is A.

Write a C++/C/Java program that solves for the depth y from $t = 0$ to 5 sec with a step size of 0.1 sec. Use the parameter values $A = 1200\ \text{m}^2$ and $Q = 400\ m^3/\text{sec}$. The depth is zero at $t = 0$. Your code has to output an ASCII text file, named `output.csv`, that contains 50 numbers separated by commas representing how the water depth changes over time.

Suppose this text file is stored as `c:\output.csv`, read the file into Mathcad by using a command, `READPRN("C:\output.csv")`, and plot a graph. You can download and study a sample text file, `output.csv`, and a Mathcad script, `readprn.mcd`, from the schedule section of the class web page.

In your hand-in directory on AFS (see the course information section of the class web for the actual location on AFS), make a new directory called `ps2-1` (in lower case). Hand in the following in your hand-in directory.

- Source code files and header files
- Executable file
- Output text file, `output.csv`

Also hand in a printout of the following:

- Source code files and header files
- Output text file
- Mathcad file with a graph

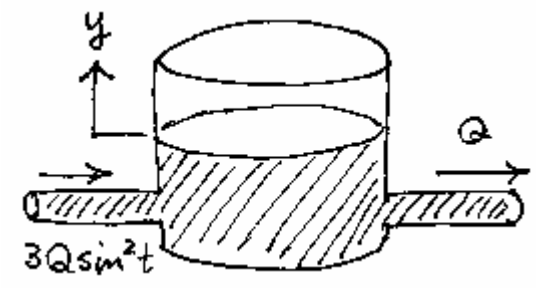

Figure 1. A storage tank containing a liquid

PS2-2 Taylor Series Approximation

Use zero-, first-, second-, and third-order Taylor series expansions to predict $f(2)$

$$f(x) = 25x^3 - 6x^2 + 7x - 88$$

using a base point at $x = 1$. Calculate the error for each of the Taylor series expansions.
error = (true value – approximated value by Taylor series) / (true value)

PS2-3 Finding Roots of Equations with Mathcad

Real mechanical systems may involve the deflection of nonlinear springs. As shown in Figure 2, a block of mass m is released a distance h above a nonlinear spring. The resistance force F of the spring is given by $F = -\left(k_1 d + k_2 d^{3/2}\right)$. Conservation of energy can be used to show that

$$f(d) = \frac{1}{2}k_1 d^2 + \frac{2}{5}k_2 d^{5/2} - mgd - mgh = 0$$

Solve the above equation for d, given the following parameter values: $k_1 = 40{,}000$ g/s^2, $k_2 = 40$ g/(s$^2 m^{0.5}$), $m = 95$ g, $g = 9.8$ m/s^2, and $h = 0.43$ m. Plot the function $f(d)$ and find the root by using the `root` function of Mathcad. Name this Mathcad file `ps2-3.mcd` and copy it to your hand-in AFS directory, `ps2-3` (in lower case). Also, hand in the printout of the Mathcad file. (Include the title of this problem and your name, e.g., "Maximum Deflection of a Non-Linear Spring by Kenji Shimada," at the top of your Mathcad file.)

To learn how to use the `root` command in Mathcad, download and study a sample Mathcad file, `find_root.mcd`, from the schedule section of the class web page.

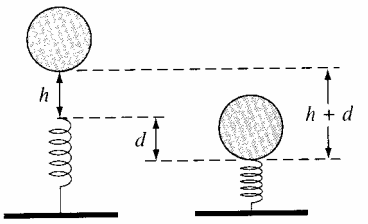

Figure 2: Nonlinear spring

PS2-4 Bisection for Root Finding

Determine the real roots of $f(x) = -2 + 7x - 5x^2 + 8x^3$ using bisection to locate the lowest root. Employ initial guesses of $x_l = 0$ and $x_u = 1$ and iterate until the approximated relative error ε_a falls below a level of $\varepsilon_s = 10\%$.

PS2

The first letter of your LAST name

First Name _____ Last Name _____

PS2-1 (40 pts)	PS2-2 (20 pts)	PS2-3 (20 pts)	PS2-4 (20 pts)	Total (100 pts)

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