

## 24-311 NUMERICAL METHODS Fall 03

Carnegie Mellon University

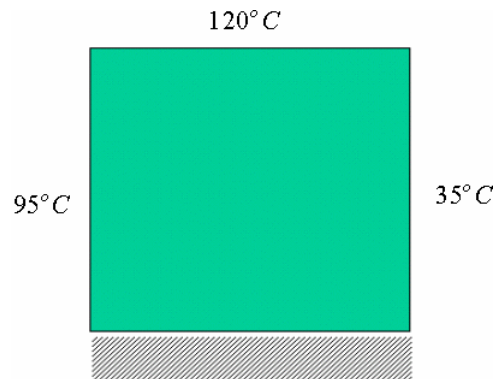
### PROBLEM SET 13

**Issued:** 11/23/03  
**Due:** 12/3/03 Wed 1:00 pm  
**Weight:** 4 % of total grade

#### Finite-Difference Solution for an Elliptic PDE

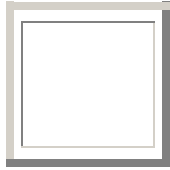
Solve for the temperature distribution of the heated square plate in the following figure using the finite-difference method. The size of the plate is 4 cm by 4 cm, and the following boundary conditions are imposed:

- The top edge of the square has a fixed temperature,  $120^\circ C$ .
- The right-hand side edge of the square has a fixed temperature,  $35^\circ C$ .
- The left-hand side edge of the square has a fixed temperature,  $95^\circ C$ .
- The bottom edge is insulated.



- (1) Define a 5 by 5 grid points over the square plate, and denote the temperature at  $x = i$  cm and  $y = j$  cm as  $T_{ij}$ . For example the temperature at the center of the plate is denoted as  $T_{22}$ . Among the 25 temperature variables at the 25 grid points, which ones are known from the boundary conditions? What are these known temperatures?
- (2) How many unknown temperatures are there? Derive the  $n$  by  $n$  matrix equation, where  $n$  is the number of unknown temperatures, to be solved for finding the steady temperature solutions.
- (3) Solve the matrix equation by using a mathematical package, and make a surface plot and a contour plot that show the temperature distribution over the plate.
- (4) What are the heat flux vectors at  $(x, y) = (2, 2)$  and  $(x, y) = (2, 0)$ ? Assume that:  $k\rho C = 1$ .

**PS13**



The first letter of  
your LAST name

\_\_\_\_\_ **First Name**

\_\_\_\_\_ **Last Name**

PS13-(1) (20 pts)	PS13-(2) (20 pts)	PS13-(3) (30 pts)	PS13-(4) (30 pts)	Total (100 pts)

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