

24-311 NUMERICAL METHODS Fall03

Carnegie Mellon University

PROBLEM SET 11

Issued: 11/07/03
Due: 11/14/03 Friday 1:00PM @ HH B127
Weight: 4 % of total grade

PS11-1 Numerical Differentiation

Taylor's series can be written as follows:

$$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \dots + \frac{f^n(x_i)}{n!}h^n + \frac{f^{n+1}(\xi)}{(n+1)!}h^{n+1}$$

- (1) Show the derivation of the following backward finite-divided-difference approximation of f'' using Taylor's series.

$$f''(x_i) = \frac{f(x_i) - 2f(x_{i-1}) + f(x_{i-2}))}{h^2} + O(h)$$

Include the error term with the "big-o" notation, i.e., $O(h)$, $O(h^2)$, $O(h^3)$, etc.

- (2) Show the derivation of the following backward finite-divided-difference approximation of f' using Taylor's series.

$$f'(x_i) = \frac{3f(x_i) - 4f(x_{i-1}) + f(x_{i-2}))}{2h} + O(h^2)$$

Include the error term with the "big-o" notation, i.e., $O(h)$, $O(h^2)$, $O(h^3)$, etc.

PS11-2 Reducing Second Order ODE to a Coupled First Order ODEs

During taxiing, an airplane experience vibration in the vertical direction due to the unevenness of the runway. By idealizing the airplane and the landing gear as a simple mass-spring-damper system as shown in Figure 1. (Note: Taxiing is the controlled movement of the airplane under its own power while on the ground.)

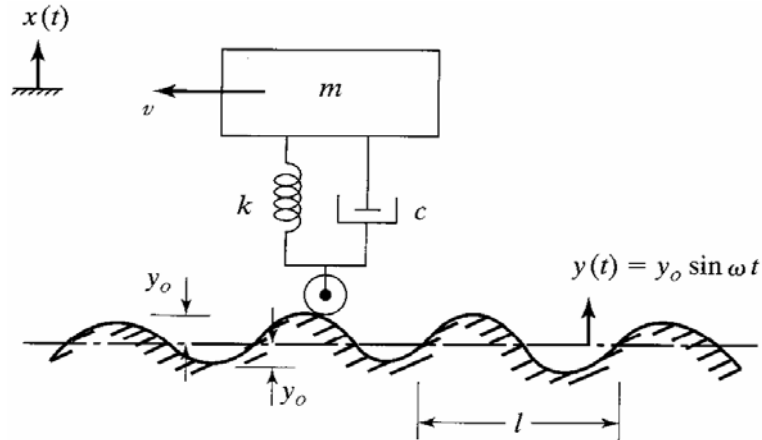


Figure 1. Airplane vertical motion along an uneven runway

- (1) What is the equation of motion of the airplane? You are interested in finding the vertical displacement of the airplane relative to the runway.

m = mass of the airplane,
 c = damping constant of the landing gear,
 k = stiffness of the landing gear,
 x = displacement of the airplane relative to the runway,
 y = runway roughness,
 v = constant taxiing speed of the airplane,
 t = time,

By modeling the runway roughness to be sinusoidal with amplitude y_0 and wavelength l and the airplane to be initially at rest state the resulting boundary-value problem.

- (2) The equation of motion that you found in the previous problem is a second order ODE. This equation can be reduced to coupled first-order equations by defining the state variables to be (x_1, x_2) , where $x_1 = x(t)$ and $x_2 = \frac{dx(t)}{dt}$. Find the system of first-order equations of the following form:

$$\begin{cases} x_1' = f_1(x_1, x_2, t) \\ x_2' = f_2(x_1, x_2, t) \end{cases}$$

PS11-3 Reducing Higher-Order ODE to First-Order ODEs

Consider the particle-damper-spring system shown in the figure below. A particle of weight m is suspended on: (1) a weightless damper of damping coefficient c ; and (2) a weightless spring of spring constant k and neutral length l_0 . The forces acting on the particle are: (1) its weight mg , (2) force due to the damper, and (3) force due to the spring. The particle moves in the xy -plane, and the position of the particle (x, y) is a function of time. You are interested in calculating the two dimensional trajectory of the particle as a function of time using numerical methods.

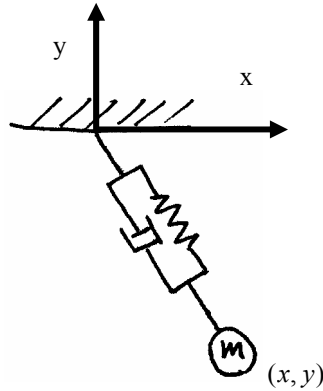


Figure 2. A -damper-spring system

- (1) The trajectory of the particle is governed by the two equations of motion in the x and y -directions. What are the two equations of motion?
- (2) The two equations of motion that you found in the previous problem are both second order ODEs. These equations can be reduced to a system of first-order equations by defining two more variables. Find the system of first-order equations.

PS11-4 C++/Java Programming—Euler’s method and Heun’s method

A storage tank contains a liquid at depth y , where $y = 0$ when the tank is half full. Liquid is withdrawn at a constant flow rate $Q = 400 \text{ m}^3/\text{sec}$ to meet demands. The contents are re-supplied at a sinusoidal rate $3Q \sin^2(t)$ (see Figure 3). The cross-sectional area of the tank is $A = 1200 \text{ m}^2$. The depth is zero at $t = 0$.

The governing ordinary differential equation of this system is:

$$\frac{dy}{dt} = 3 \frac{Q}{A} \sin^2 t - \frac{Q}{A}$$

Write a C++/Java program that solves the above first order ODE for the depth y from $t = 0$ to 10 sec with initial condition $y(0) = 0$ using two numerical methods, Euler’s and Heun’s. Your program should first prompt a user to input a step size from keyboard and then use the specified step size in the two numerical integration schemes.

Your code has to output two ASCII text files, named `euler.csv` and `heun.csv`, that contain numbers, one in each line, representing how the water depth changes over time. Use Mathcad to read the data from the two CSV files and plot the two solution trajectories, one by Euler and the other by Heun, in a single graph.

Run your code with three different time steps: 0.01 sec., 0.1 sec., and 1.0 sec.

In your hand-in directory on AFS (see the course information section of the class web for the actual location on AFS), make a new directory called `ps11-4` (in lower case). Hand in the following in your hand-in directory.

- Source code files and header files
- Executable file
- Output CSV files for the three time steps (Rename `euler.csv` to `euler01.csv`, `euler01.csv`, `euler1.csv` for the three time steps. Use the same renaming scheme for `heun.csv`.)

Also hand in a printout of the following:

- Source code files and header files
- Three Mathcad files

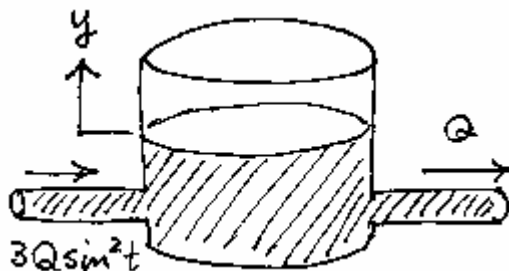
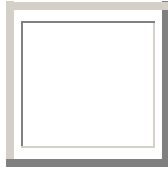


Figure 3. A storage tank containing a liquid

PS11



The first letter of
your LAST name

_____ **First Name**

_____ **Last Name**

PS11-1 (20 pts)	PS11-2 (20 pts)	PS11-3 (20 pts)	PS11-4 (40 pts)	Total (100 pts)

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