

## 24-311 NUMERICAL METHODS Fall03

Carnegie Mellon University

### PROBLEM SET 10

**Issued:** 11/1/03  
**Due:** 11/7/03 (Friday) 1:00PM @ HH B127  
**Weight:** 4 % of total grade

#### PS10-1 Piecewise Quadratic Spline Interpolation

Interpolate the following five points with quadratic splines: (0, 0), (1, 1), (2,4), (3,3), (4,1). Assume that the slopes at (0, 0) is 0.

Show all the intermediate steps, including the derivation of the matrix equation. Solve the matrix equation with Mathcad, and show the resultant spline shapes with Mathcad. (Note: do not use Mathcad's built-in function to do the Spline interpolation:-)

**PS10-2** What is quartic Lagrange's interpolation polynomial function  $f(x)$  that interpolates five points: (0,2), (1,3), (3,0), (4,4), and (6,1). Find the function value at  $x=2$  using the polynomial function.

#### PS10-3 Analytical and Numerical Integration of a Function

(1) Integrate the following function analytically. Use at least five significant digits.

$$I = \int_0^{\pi} (3 + 5 \sin x) dx$$

- (2) Integrate the function  $I$  using a single application of the trapezoidal rule. Compute the percent relative error for the numerical result.
- (3) Integrate the function  $I$  using Simpson's 1/3 rule. Compute the percent relative error for the numerical result.
- (4) Integrate the function  $I$  using Simpson's 3/8 rule. Compute the percent relative error for the numerical result.

#### PS10-4 Derivation of Simpson's 1/3 Rule using Quadratic Lagrange Interpolating Polynomial

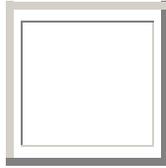
In class we derived the trapezoidal rule using the linear Lagrange interpolating polynomial. Using a similar approach, derive Simpson's 1/3 rule

$$I = (x_2 - x_0) \frac{f(x_0) + 4f(x_1) + f(x_2)}{6}$$

by integrating the quadratic Lagrange interpolating polynomial

$$f_2(x) = L_0 f(x_0) + L_1 f(x_1) + L_2 f(x_2).$$

**PS10**



The first letter of  
your LAST name

\_\_\_\_\_ **First Name**

\_\_\_\_\_ **Last Name**

PS10-1 (25 pts)	PS10-2 (25 pts)	PS10-3 (25 pts)	PS10-4 (25 pts)	Total (100 pts)

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