Quiz 5

The first letter of
your LAST name

First Name

Last Name

Q5-1 (25 pts)	Q5-2 (25 pts)	Q5-3 (25 pts)	Q5-4 (25 pts)	Total

Note: You have 75 min. Be careful about the time allocation. Try not to leave any problems totally blank so that I can give you some partial credit! Good luck.

24-311 NUMERICAL METHODS Fall 02

QUIZ 5

Date and time	**12/5 (Thr), 10:35-11:50PM (75 min)**
Weight	**10% of final grade**
Coverage	**Topics** **Numerical Integration of ODE** **Numerical Integration of PDE** **Problem Sets** **PS11, PS12, and PS13** **Lectures and Handouts** **24-28**
Format	**closed book, closed notes**
Note	**bring a basic calculator**

Q5-1 (1) Are the following ODEs linear or nonlinear? Circle all the linear ODEs in the following list of equations. (Note: y' is the first derivative of y with respect to x, and y'' the second derivative.)

(25 pts)

$$y'x^2 + yx + 1 = 0 \qquad y' = x^2 y + x^3$$

$$y'' = \frac{y'}{x^2} \qquad y' = y\sin(x)$$

$$\qquad y' = x\sin(y)$$

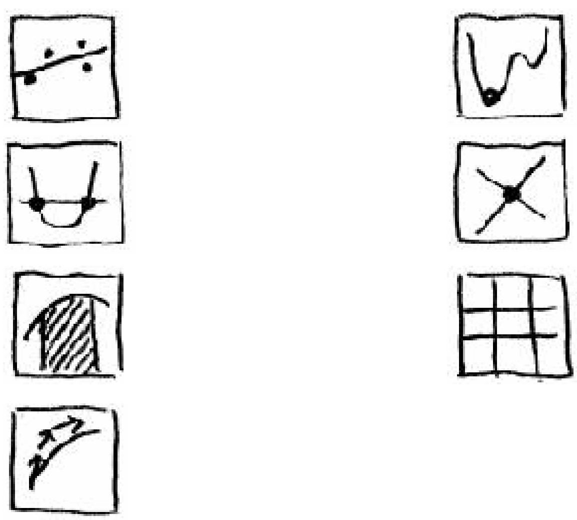
$$y'' = xy^{-1} \qquad \sqrt{x}\,y'' + y' + x = 0$$

(2) Are the following PDEs linear or nonlinear? Circle all the linear PDEs in the following list of equations.

$$\frac{\partial^2 u}{\partial x^2} + \sin(xy)\frac{\partial^2 u}{\partial y^2} = u \qquad u\frac{\partial^3 u}{\partial x^2 \partial y} + \frac{\partial^2 u}{\partial x \partial y} = 1$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \frac{1}{u} \qquad \frac{\partial^3 u}{\partial x^2 \partial y} + \log(x+y)\frac{\partial^2 u}{\partial x \partial y} = u$$

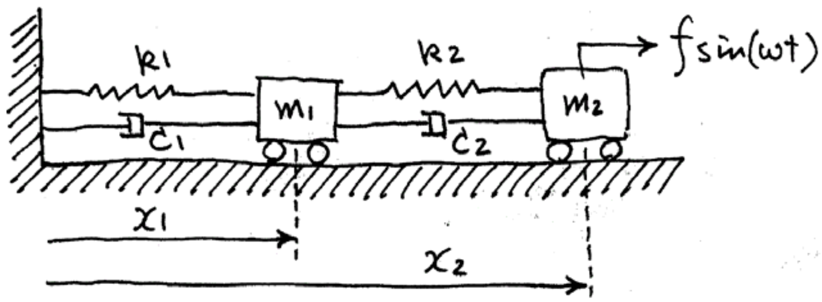
(3) The icons below represent seven topics that we covered in class. Write the name of each topic on the right hand side of the icons.



(4) The above seven topics are closely related. For example, a solution to a certain type of PDE can be obtained by solving a matrix equation. Name two more examples of such relationships between topics.

Q5-2 You are interested in finding the behavior of the mass-damper-spring system shown in the picture below. The system consists of two masses, two dampers, and two linear springs. The weights of the masses are m_1 and m_2, the damping coefficients are c_1 and c_2, the spring constants are k_1 and k_2, and the neutral length of the spring are l_1 and l_2 respectively. An external force, $f\sin(\omega t)$, is applied to the second mass.

(25 pts)



(1) Find a set of ODEs, or governing equations of motion, of the system.

(2) The set of governing equations of motion that you found in the previous problem are second order ODEs. These equations need to be reduced to a set of coupled first-order equations for applying a Runge-Kutta method. What are the set of coupled first-order equations?

Q5-3 Consider the following first order ODE. Integrate the ODE using the mid-point method and the fourth order Runge-Kutta method with an initial condition, $y = 1$ at $x = 1$, and the interval of numerical integration, $h = 2$.

(25 pts)

$$y' + 2xy - x^2 = 0$$

(1) Integrate the ODE using the mid-point method once.
What are k_1 and k_2 ?
What is the representative slope, ϕ ?
What is the value of y after one iteration, i.e., y (3)?

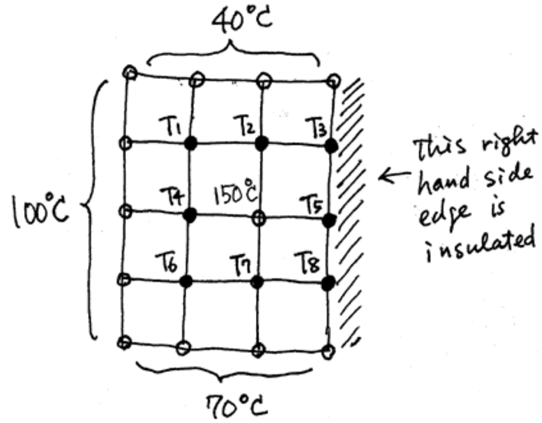
(2) Integrate the same ODE using the fourth-order Runge-Kutta method once.
What are k_1, k_2, k_3, and k_4?
What is the representative slope, ϕ?
What is the value of y after one iteration, i.e., $y(3)$?

Q5-4 The steady-state temperature distribution of a plate can be represented by an elliptic partial differential equation called the Laplace equation:

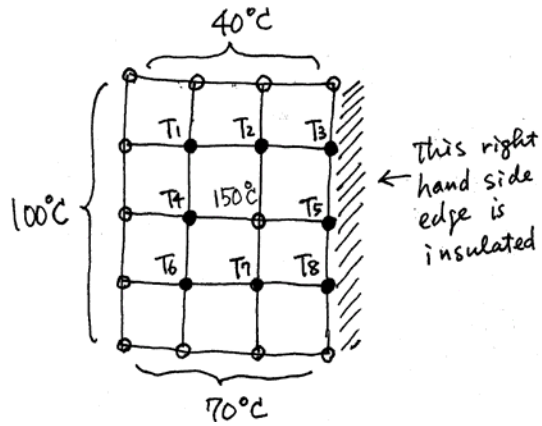
(25 pts)

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$$

Suppose you define four by five grid points and specify the temperatures on the boundary and one internal point as shown in the picture below.



Find the eight by eight matrix equation to be solved in order to find the steady-state temperature distribution of a plate, T_1, T_2, T_3, T_4, T_5, T_6, T_7, and T_8.


40°C
100°C
70°C
T1
T2
T3
T4
150°C
T5
T6
T7
T8
This right hand side edge is insulated

This is the last page of Quiz 5, and this page is intentionally left blank
so that you can use it if you need more space to write your solution or do some calculations.