

3.2 Gauss Elimination

"naive" Gauss Elimination

= two-step procedures (elimination + back substitution)

$$\begin{cases} x_1 + x_2 - x_3 = -3 \\ 6x_1 + 2x_2 + 2x_3 = 2 \\ -3x_1 + 4x_2 + x_3 = 1 \end{cases} \xrightarrow{\text{matrix form}} \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix} \begin{Bmatrix} \\ \\ \end{Bmatrix} = \begin{Bmatrix} \\ \\ \end{Bmatrix}$$

or

$$\begin{array}{cccc} 1 & 1 & -1 & -3 & \text{--- ①} \\ 6 & 2 & 2 & 2 & \text{--- ②} \\ -3 & 4 & 1 & 1 & \text{--- ③} \end{array}$$

Elimination step

Second row:

$$\begin{array}{cccc} 1 & 1 & -1 & -3 & \text{--- ④} \\ & & & & \text{--- ⑤} \\ & & & & \text{--- ⑥} \end{array}$$

Third row:

Third row:

$$\begin{array}{cccc} 1 & 1 & -1 & -3 & \text{--- ⑦} \\ & & & & \text{--- ⑧} \\ & & & & \text{--- ⑨} \end{array}$$

Back-substitution step

Find x_3 from ④

Find x_2 from ⑧

Find x_1 from ⑦

Pitfalls of Naive G.E.

- ① Division by zero
- ② Round-off errors
- ③ Ill-conditioned systems.
- ④ Singular systems.

G.E. with partial pivoting

To reduce round-off errors caused by division by a small pivot coefficient

Switch rows so that the largest number is used as the pivot coeff.

Elimination step

1	1	-1	-3	— ①
6	2	2	2	— ②
-3	4	1	1	— ③

Pivoting (① ↔)

— ④

— ⑤

— ⑥

Second row:

Third row:

— ⑦

— ⑧

— ⑨

Pivoting (⑧ ↔)

— ⑩

— ⑪

— ⑫

Third row:

— ⑬

— ⑭

— ⑮

Back-substitution
step

Find x_3 from (15)

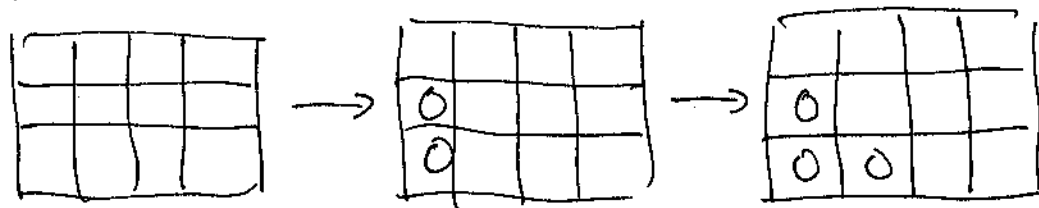
Find x_2 from (14)

Find x_1 from (15)

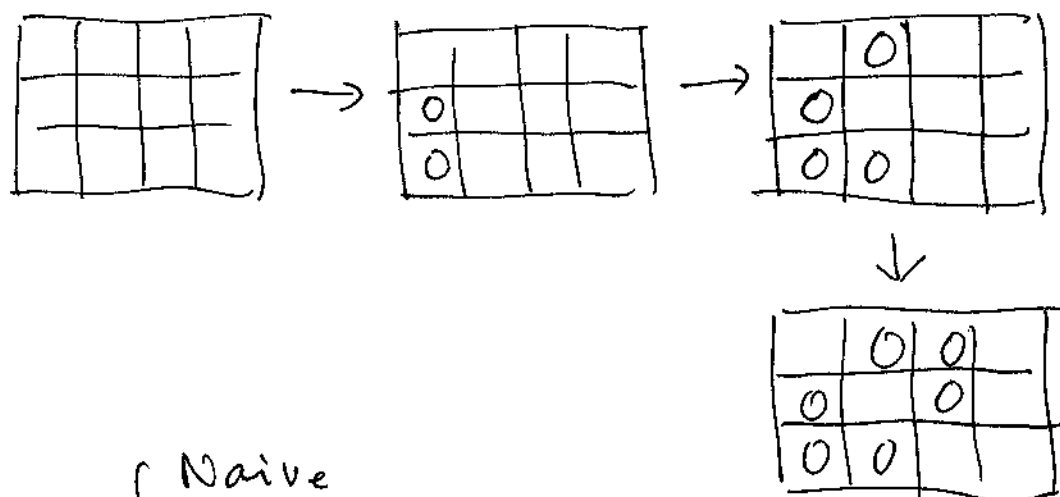
⊗ Read 9.4.3 scaling (normalization)
↑
to reduce round-off errors

The Gauss - Jordan Method

G.E.



G.J.



G.J. { Naive
with partial pivoting
with scaling (normalization)