# 3.2 Gauss Elimination

## "naive" Gauss Elimination

= two-step procedures ( elimination + back substitution)

$$\begin{cases} x_1 + x_2 - x_3 = -3 \\ 6x_1 + 2x_2 + 2x_3 = 2 \\ -3x_1 + 4x_2 + x_3 = 1 \end{cases} \xrightarrow{\text{matrix form}} \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{Bmatrix} \\ \\ \end{Bmatrix} = \begin{Bmatrix} \\ \\ \end{Bmatrix}$$

or

$$\begin{array}{rrrrl} 1 & 1 & -1 & -3 & \text{— ①} \\ 6 & 2 & 2 & 2 & \text{— ②} \\ -3 & 4 & 1 & 1 & \text{— ③} \end{array}$$

### Elimination step

Second row:

$$\begin{array}{rrrrl} 1 & 1 & -1 & -3 & \text{— ④} \\ & & & & \text{— ⑤} \\ & & & & \text{— ⑥} \end{array}$$

Third row:

Third row:

$$\begin{array}{rrrrl} 1 & 1 & -1 & -3 & \text{— ⑦} \\ & & & & \text{— ⑧} \\ & & & & \text{— ⑨} \end{array}$$

## Back-substitution step

Find $x_3$ from (9)

Find $x_2$ from (8)

Find $x_1$ from (7)

## Pitfalls of Naive G.E.

(1) Division by zero

(2) Round-off errors

(3) Ill-conditioned systems.

(4) Singular systems.

## G. E. with Partial Pivoting

To reduce round-off errors caused by division by a small pivot coefficient
Switch rows so that the largest number is used as the pivot coeff.

Elimination step

| | | | | |
|---|---|---|---|---|
| 1 | 1 | −1 | −3 | — ① |
| 6 | 2 | 2 | 2 | — ② |
| −3 | 4 | 1 | 1 | — ③ |

Pivoting ( ① ↔ )

— ④

— ⑤

— ⑥

Second row: Third row:

— ⑦

— ⑧

— ⑨

Pivoting ( ⑧ ↔ )

— ⑩

— ⑪

— ⑫

Third row:

— ⑬

— ⑭

— ⑮

Back-substitution step

Find $x_3$ from (15)

Find $x_2$ from (14)

Find $x_1$ from (15)

(*) Read 9.4.3 scaling (normalization)

to reduce round-off errors

# The Gauss - Jordan Method

G. E.

$$
\begin{bmatrix} \ & \ & \ & \ \\ \ & \ & \ & \ \\ \ & \ & \ & \ \end{bmatrix}
\longrightarrow
\begin{bmatrix} \ & \ & \ & \ \\ 0 & \ & \ & \ \\ 0 & \ & \ & \ \end{bmatrix}
\longrightarrow
\begin{bmatrix} \ & \ & \ & \ \\ 0 & \ & \ & \ \\ 0 & 0 & \ & \ \end{bmatrix}
$$

G. J.

$$
\begin{bmatrix} \ & \ & \ & \ \\ \ & \ & \ & \ \\ \ & \ & \ & \ \end{bmatrix}
\longrightarrow
\begin{bmatrix} \ & \ & \ & \ \\ 0 & \ & \ & \ \\ 0 & \ & \ & \ \end{bmatrix}
\longrightarrow
\begin{bmatrix} \ & 0 & \ & \ \\ 0 & \ & \ & \ \\ 0 & 0 & \ & \ \end{bmatrix}
\longrightarrow
\begin{bmatrix} \ & 0 & 0 & \ \\ 0 & \ & 0 & \ \\ 0 & 0 & \ & \ \end{bmatrix}
$$

$$
\text{G. J} \begin{cases} \text{Naive} \\ \text{with partial pivoting} \\ \text{with scaling (normalization)} \end{cases}
$$