FIG. 16

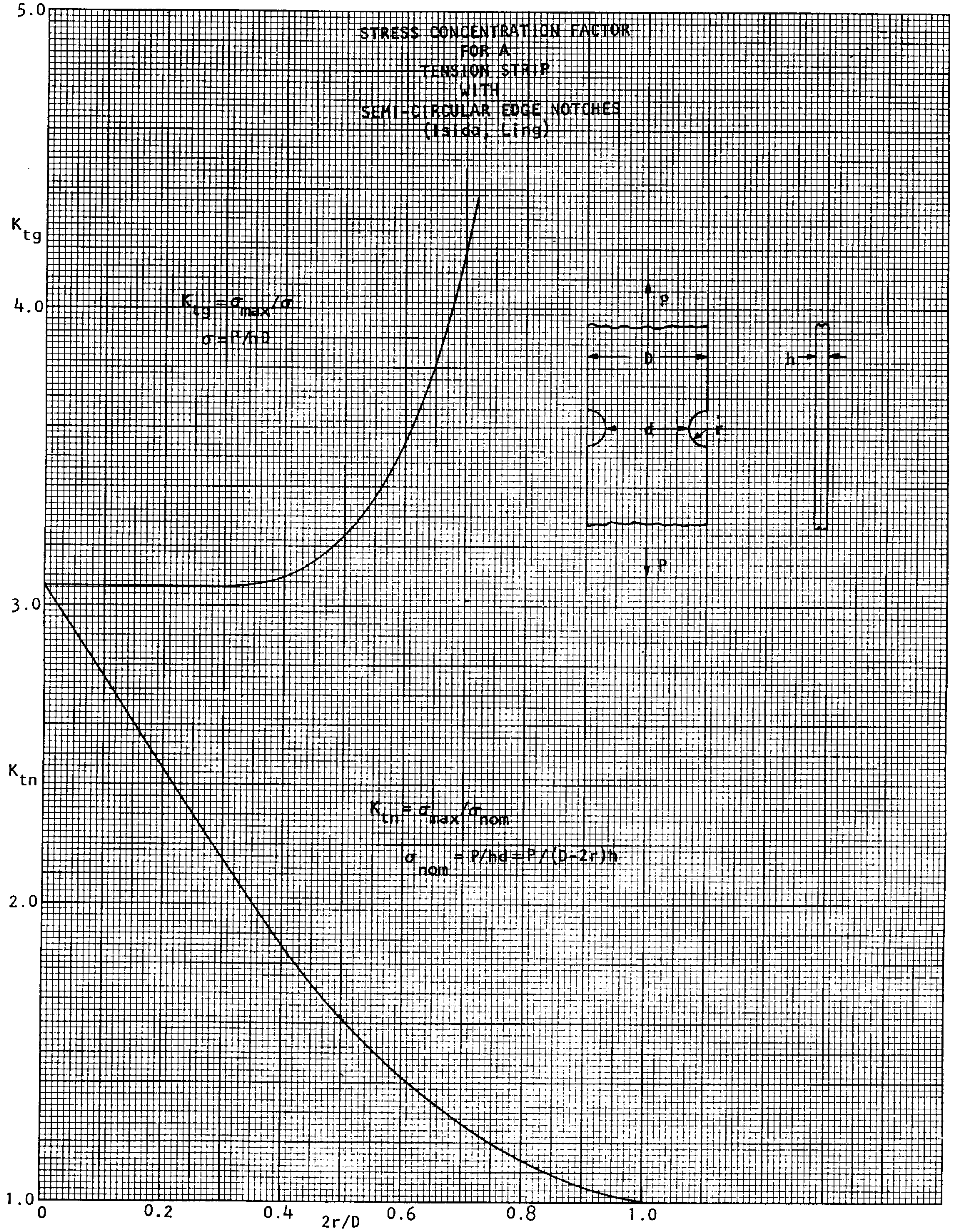

STRESS CONCENTRATION FACTOR
FOR A
TENSION STRIP
WITH
SEMI-CIRCULAR EDGE NOTCHES
(Isida, Ling)
5.0
4.0
3.0
2.0
1.0
K_{tg}
K_{tn}
$K_{tg} = \sigma_{max}/\sigma$
$\sigma = P/hD$
$K_{tn} = \sigma_{max}/\sigma_{nom}$
$\sigma_{nom} = P/hd = P/(D-2r)h$
P
D
d
r
h
0
0.2
0.4
2r/D
0.6
0.8
1.0

FIG. 48

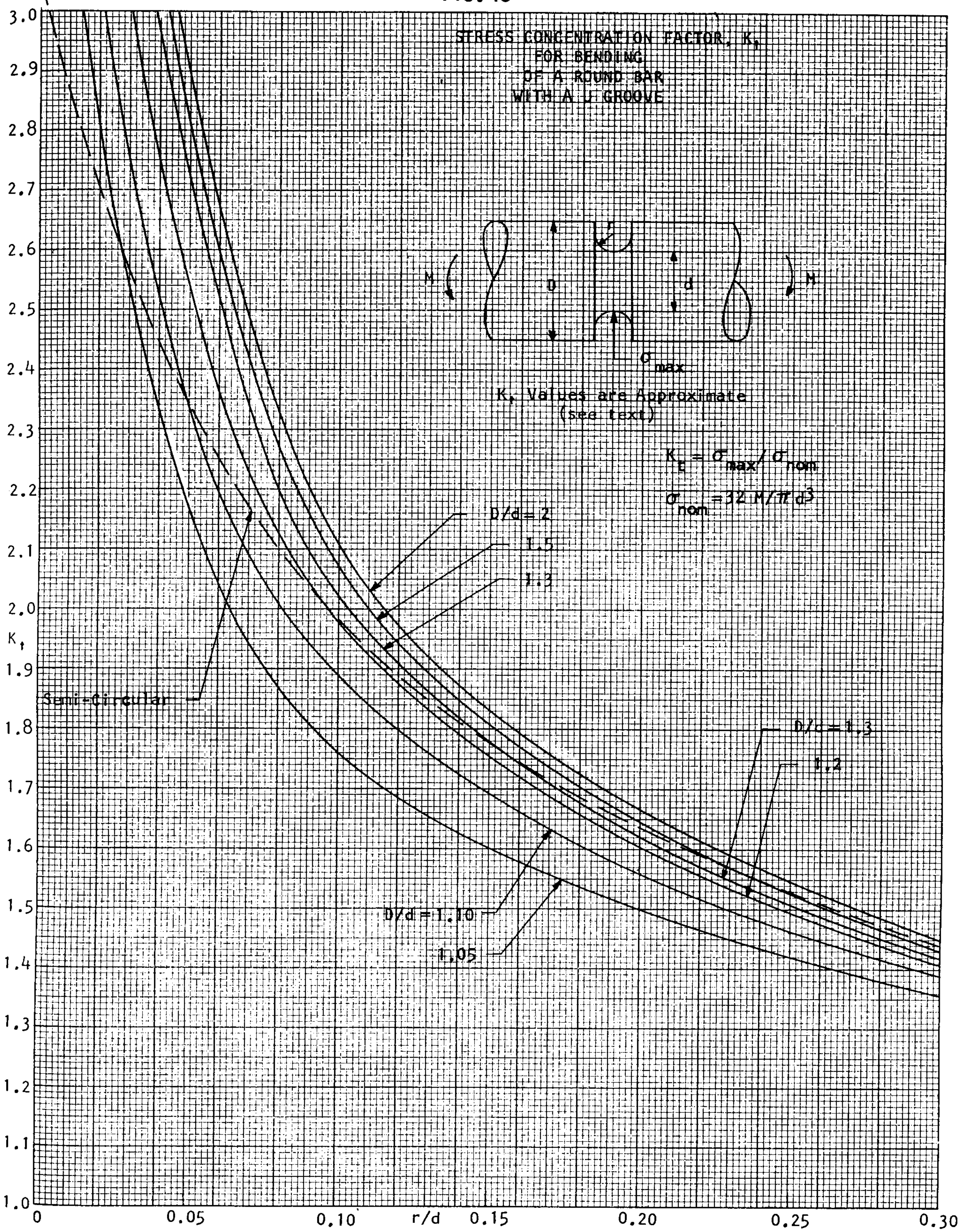

STRESS CONCENTRATION FACTOR, K_t
FOR BENDING
OF A ROUND BAR
WITH A U GROOVE
M
D
r
d
M
σ_{max}
K_t Values are Approximate
(see text)
$K_t = \sigma_{max}/\sigma_{nom}$
$\sigma_{nom} = 32\,M/\pi d^3$
D/d = 2
1.5
1.3
Semi-Circular
D/d = 1.3
1.2
D/d = 1.10
1.05
K_t
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0
0.05
0.10
r/d
0.15
0.20
0.25
0.30

FIG. 54

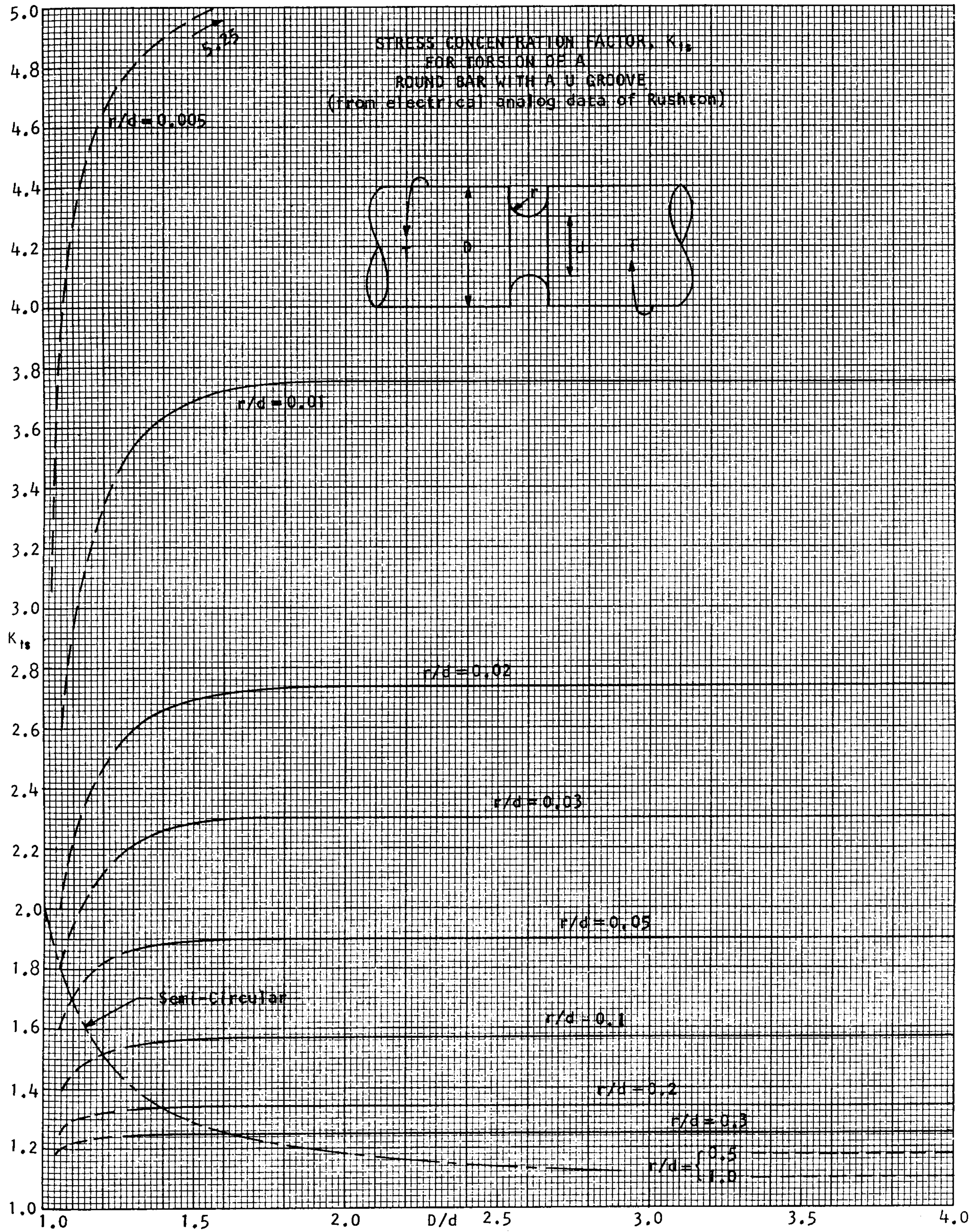
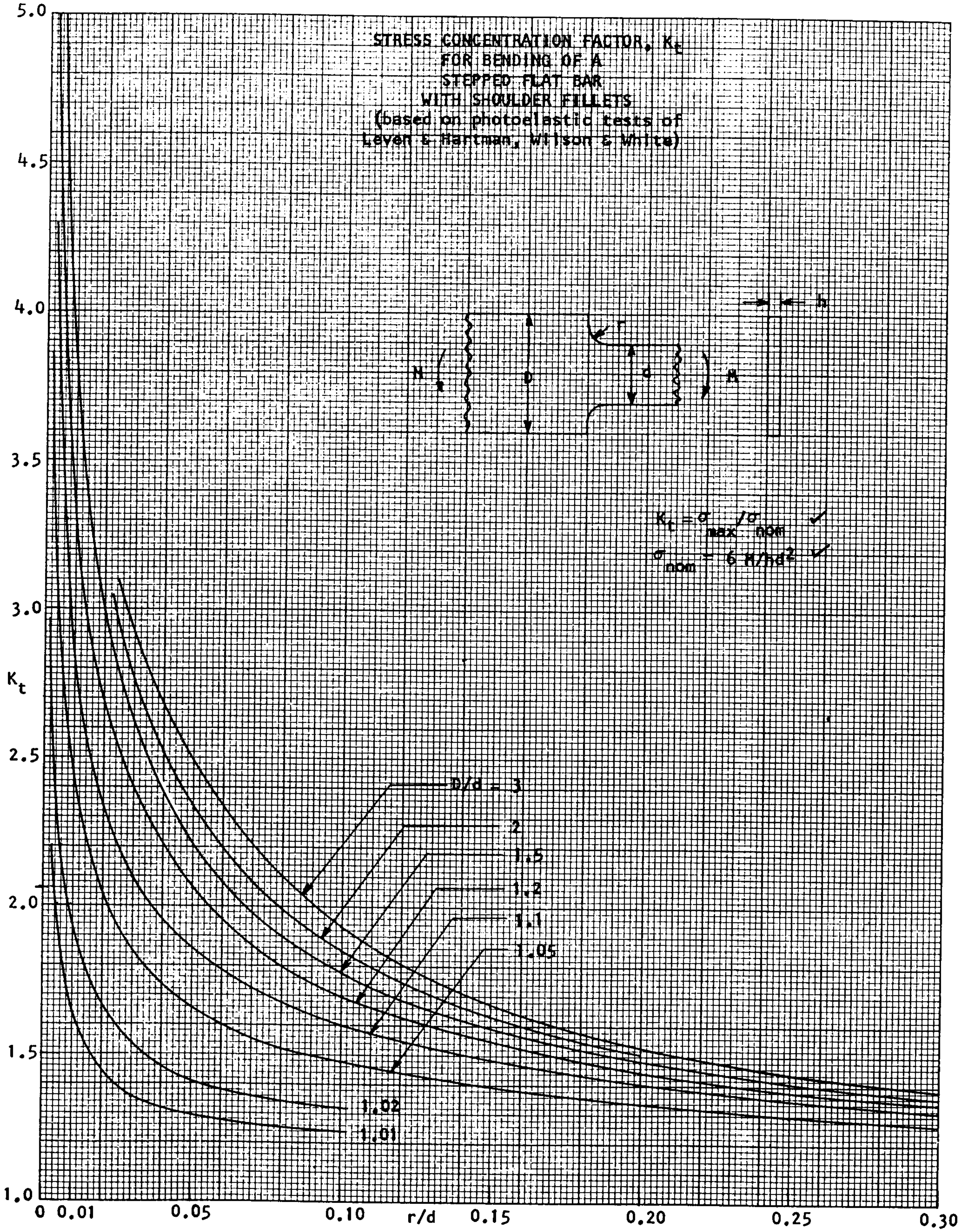

STRESS CONCENTRATION FACTOR, K_{ts}
FOR TORSION OF A
ROUND BAR WITH A U GROOVE
(from electrical analog data of Rushton)
r/d = 0.005
r/d = 0.01
r/d = 0.02
r/d = 0.03
r/d = 0.05
r/d = 0.1
r/d = 0.2
r/d = 0.3
r/d = {0.5, 1.0}
Semi-Circular
K_{ts}
D/d
D
d
r
T
5.0
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
1.0
1.5
2.0
2.5
3.0
3.5
4.0

FIG. 73


STRESS CONCENTRATION FACTOR, K_t
FOR BENDING OF A
STEPPED FLAT BAR
WITH SHOULDER FILLETS
(based on photoelastic tests of
Leven & Hartman, Wilson & White)
h
r
M
D
d
M
$K_t = \sigma_{max}/\sigma_{nom}$
$\sigma_{nom} = 6\ M/hd^2$
D/d = 3
2
1.5
1.2
1.1
1.05
1.02
1.01
K_t
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0
0.01
0.05
0.10
r/d
0.15
0.20
0.25
0.30

It would seem that a rounded corner at the intersection (in the plane of the hole axes) would be beneficial in reducing K_t. This would be a practical expedient in the case of a tunnel or a cast metal part, but it does not seem to be practically attainable in the case where the holes have been drilled. An investigation of three-dimensional photoelastic models with the corner radius varied would be of interest.

For pressurized thick-walled cylinders with crossholes and sideholes, see Chapter 5, Section Q.

(B) BENDING

For thin plates or beams, two kinds of bending are presented: in-plane bending, Sections B.a, B.b, and B.c; transverse bending, Sections B.d, B.e, B.f, and B.g. For transverse bending, two cases are considered: simple bending ($M_1 = 1$, $M_2 = 0$) and cylindrical bending ($M_1 = 1$, $M_2 = \nu$). The cylindrical bending case removes the anticlastic bending resulting from the Poisson's ratio effect. At the beginning of application of bending, the simple condition occurs. As the deflection increases, the anticlastic effect is not realized, except for a slight curling at the edges. In the region of the hole, it is reasonable to assume that the cylindrical bending condition exists. For design problems, the cylindrical bending case is generally more applicable than the simple bending case.

It would seem that for transverse bending, rounding or chamfering of the hole edge would result in reducing the stress concentration factor.

For $M_1 = M_2$, isotropic transverse bending, K_t is independent of a/h, diameter of hole over thickness of plate; the case corresponds to biaxial tension of a plate with a hole.

(a) In-Plane Bending of a Beam with a Central Circular Hole

An effective method of weight reduction for a beam in bending is to remove material near the neutral axis, often in the form of a circular hole, or a row of circular holes.

Howland and Stevenson[288] have obtained mathematically the K_{tg} values represented by the curve of Fig. 156:

$$K_{tg} = \frac{\sigma_{max}}{6M/w^2t} \qquad [111]$$

Symbols are defined in Fig. 156. K_{tg} is the ratio of σ_{max} to σ at the beam edge distant axially from the hole.

Photoelastic tests by Ryan and Fischer[289] and by Frocht and Leven[246] are in good agreement with the mathematical results.[288]

K_{tn} is based on the section modulus of the net section; the distance from the neutral axis is taken as $a/2$, so that σ_{nom} is at the edge of the hole:

$$K_{tn} = \frac{\sigma_{max}}{6Ma/(w^3 - a^3)t} \qquad [112]$$

Another form of K_{tn} has been used where σ_{nom} is at the edge of the beam:

$$K'_{tn} = \frac{\sigma_{max}}{6Mw/(w^3 - a^3)t} \qquad [113]$$

Udoguti[290] and Heywood[290a] noted that K'_{tn} versus a/w is a linear relation, $K'_{tn} = 2a/w$. Heywood[290a] further noted that $K_{tn} = 2$, commenting that this provides the "curious result that the stress concentration factor is independent of the relative size of the hole, and forms the only known case of a notch showing such independency."

Note from Fig. 156 that the hole does not weaken the beam for $a/w < \backsim 0.45$ (for design, $K_{tg} = 1$ for $a/w < \backsim 0.45$).

On the outer edge, the stress has peaks at F, F, but this stress is less than at E, except at and to the left of a transition zone in the region of C where $K_t = 1$ is approached. Angle $\alpha = 30°$ was found to be independent of $a/(w - a)$ over the range investigated.

(b) In-Plane Bending of a Beam with a Circular Hole Displaced from the Center Line

The K_t factor, as defined by [111], has been obtained by Isida[291] and is shown in Fig. 157. At line A—A, $K_{tgB} = K_{tgC}$, corresponding to maximum stress at B and C, respectively (see sketch in Fig. 157). Above A—A, K_{tgB} is the greater of the two stresses; below A—A, K_{tgC} is the greater, approaching $K_t = 1$, or no effect of the hole.

At $c/e = 1$, the hole is central, with factors as given in the preceding section. For $r/c \to 0$, K_{tg} is 3 multiplied by the ratio of the distance from the center line to the edge; in terms of c/e:

$$K_{tg} \doteq 3\,\frac{1 - c/e}{1 + c/e} \qquad [114]$$

Photoelastic results of Nisida[292] are in agreement with the calculated values of Isida.[291]

(c) In-Plane Bending of a Beam with an Elliptical Hole; Slot with Semicircular Ends (Ovaloid); or Round-Cornered Square Hole

K'_{tn} factors, as defined by relation [113], were obtained by Isida;[169] these factors have been recalculated for K_{tg}, relation [111], and for K_{tn}, relation [112], and are presented in Fig. 158. The photoelastic values of Frocht and Leven[246] for a slot with semicircular ends are in reasonably good agreement when compared with an ellipse having the same a/r.

Note in Fig. 158 that the hole does not weaken the beam for a/w values less than at points A, B, and C for $a/r = 8$, 4, and 2, respectively (for design, $K_t = 1$ to the left of the intersection points).

On the outer edge, the stress has peaks at F,F, but this stress is less than at E, except at and to the left of a transition zone in the region of A, B, or C, where $K_t = 1$ is approached. In the photoelastic tests,[246] $\alpha = 35°$, 32.5° and 30° for $a/r = 8$, 4, and 2, respectively, independent of $a/(w - a)$ over the range investigated.

For shapes approximating ovaloids and round-cornered square holes (parallel and at 45°), K'_{tg} factors have been obtained[293] for central holes, small compared to the beam depth:

$$K'_{tg} = \frac{\sigma_{\max}}{6Ma/w^3t} \qquad [115]$$

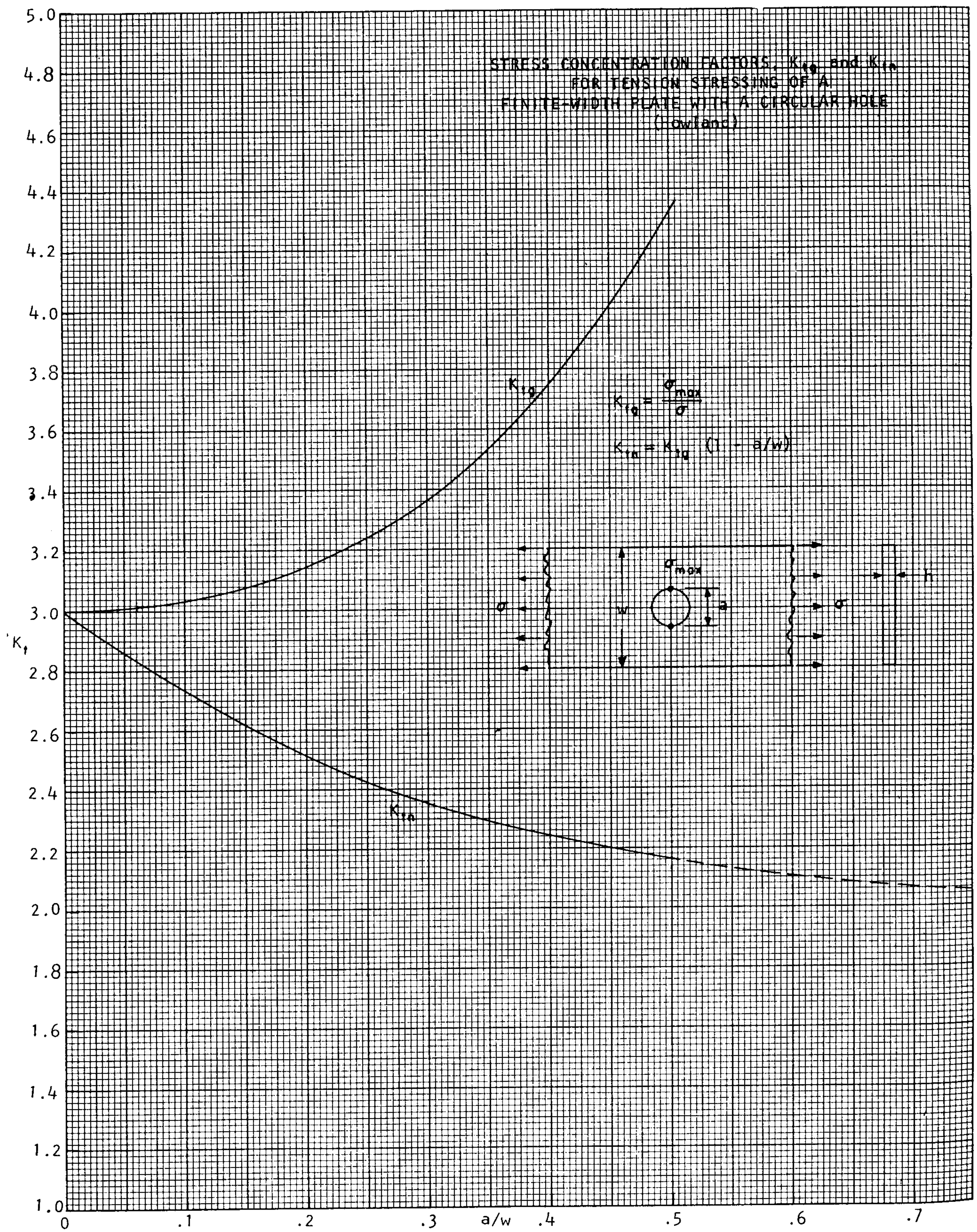
(d) Transverse Bending of an Infinite and of a Finite-Width Plate with a Single Circular Hole

For simple bending ($M_1 = 1$, $M_2 = 0$), Reissner[294] obtained K_t as a function of a/h, as shown in Fig. 159. For $a/h \to 0$, $K_t = 3$. For $a/h \to \infty$:

$$K_t = \frac{5 + 3\nu}{3 + \nu} \qquad [116]$$

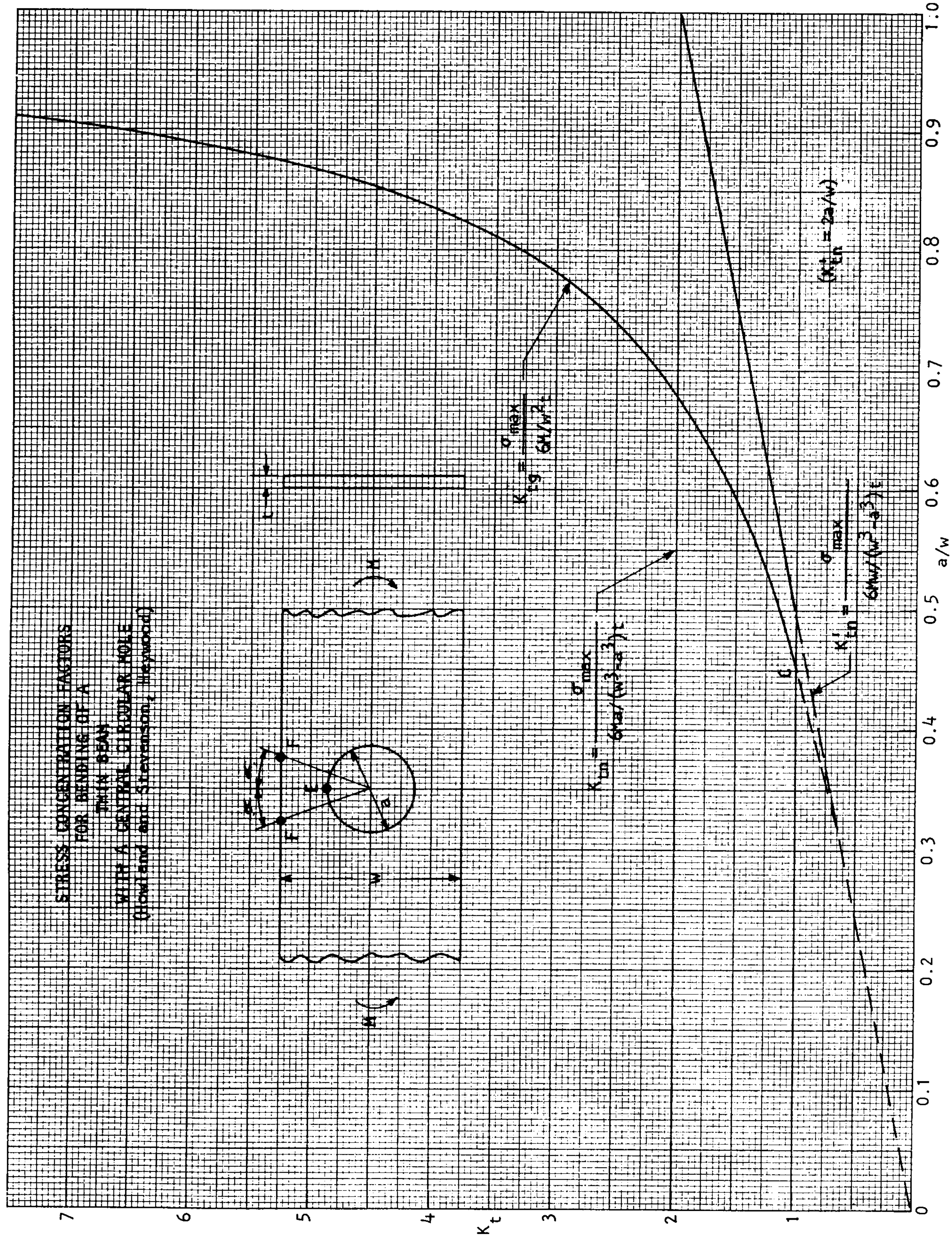
For $\nu = 0.3$, $K_t = 1.788$.

FIG. 86



STRESS CONCENTRATION FACTORS, K_{tg} and K_{tn}
FOR TENSION STRESSING OF A
FINITE-WIDTH PLATE WITH A CIRCULAR HOLE
(Howland)
$K_{tg} = \frac{\sigma_{max}}{\sigma}$
$K_{tn} = K_{tg}\ (1 - a/w)$
K_{tg}
K_{tn}
σ
σ_{max}
w
a
h
K_t
a/w
5.0
4.8
4.6
4.4
4.2
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
.1
.2
.3
.4
.5
.6
.7

FIG. 156



STRESS CONCENTRATION FACTORS
FOR BENDING OF A
THIN BEAM
WITH A CENTRAL CIRCULAR HOLE
(Howland and Stevenson; Heywood)
$K_{tg} = \frac{\sigma_{max}}{6M/w^2 t}$
$K_{tn} = \frac{\sigma_{max}}{6Ma/(w^3-a^3)t}$
$K'_{tn} = \frac{\sigma_{max}}{6Mw/(w^3-a^3)t}$
$(K'_{tn} = 2a/w)$
K_t
a/w