Contact Stresses

This brief explanation considers the very common situation of two curved bodies pressing against one another. Initially, such bodies meet at one point, but as the load pressing them together increases, the area of contact grows because of the deformation of the bodies.

The subject of contact mechanics is a complex one, and we will offer some useful results for only two important cases. An excellent reference on this subject is *Contact Mechanics* by K.L. Johnson, which is published by Cambridge University Press.

The situation of interest is depicted in the Figure 1. This actually represents many possible situations. It could be two cylinders pressing against one another. It could be two spheres pressing against one another. It could be a cylinder or a sphere pressing against a flat surface (if R_1 or R_2 is set to = ∞ in the subsequent formulas). It could be cylinder pressing against the inside of another cylinder (a bolt in a hole larger than the bolt diameter (in which case, the hole radius will be defined as negative below). We cannot handle a cylinder pressing against a sphere with the formulas below, but more complex formulas could be found.

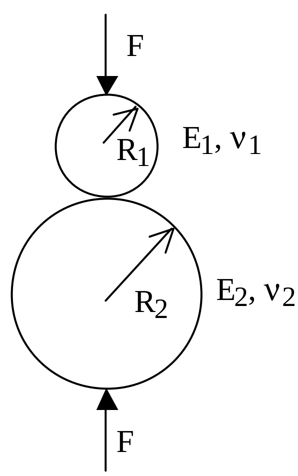

Figure 1

In all these cases of contact, there is a region of contact on which the bodies are directly pressing on one another. This is shown in Figure 2. The region of contact would only be flat if you pressed two identical cylinders against one another or two identical spheres against one another. Whether or not the contact region is flat, there will be a distribution of contact pressure between the contacting bodies. The contact pressure has a maximum value at the central point, and decreases to zero at the edge of contact. A schematic plot of the contact pressure versus position along the contact region is also shown in Figure 2.

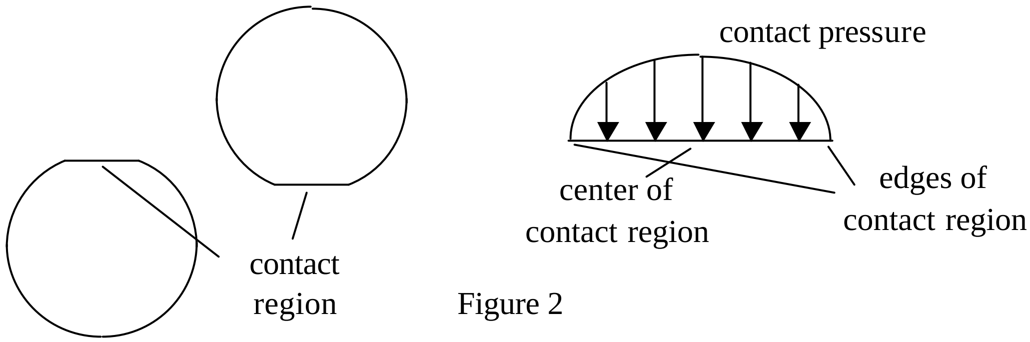

Figure 2

We denote the maximum contact pressure as p_0. Formulas for p_0 as a function of the various parameters are now given.

For cylinders in contact: $p_0 = \sqrt{\frac{FE*}{pR*}}$, where F is in units of force//length. The length is the length of contact along the direction of the cylinder axis.

For spheres in contact: $p_0 = \left[\frac{6FE*^2}{p^3 R*^2}\right]^{1/3}$, where F is in units of force.

E* and R* are related to the values of E and R for the two bodies:

$$\frac{1}{E*} = \frac{1-n_1^2}{E_1} + \frac{1-n_2^2}{E_2}$$

$$\frac{1}{R*} = \frac{1}{R_1} + \frac{1}{R_2}$$

Notice that the maximum contact pressure increases as the material becomes stiffer, because the force is transmitted over a smaller area. Likewise smaller radii lead to smaller areas and larger contact pressures.

It turns out that the maximum shear stress (which is one way of determining whether a ductile material will yield plastically) occurs some distance beneath the surface. The value of this maximum shear stress is approximately $\tau_{max} = 0.3p_0$.

Here is an example of using the equations above.

A steel roller 4 cm in diameter presses against a flat steel surface. A force of 3000 N presses the roller into the surface. What is the maximum shear stress acting in the bodies?

Assume that the contact between the cylinder and the surface is uniform along the 3 cm length.

The force per unit length pressing them together is 3000/.03 = 10^5 N/m. While the cylinder is longer, the contact is only over the 3 cm width of the flat surface.

Both materials are steel, with moduli $E_1 = E_2 = 200$ GPa and $\nu_1 = \nu_2 = 0.3$.

Therefore,

$$\frac{1}{E^*} = \frac{1-n_1^2}{E_1} + \frac{1-n_2^2}{E_2} = \frac{1-(0.3)^2}{200x10^9} + \frac{1-(0.3)^2}{200x10^9} \Rightarrow E^* = 1.10x10^{11}$$

The cylinder has a radius of $R_1 = 2$ cm = 0.02 m. The surface has a radius $R_2 = \infty$. (It does not matter which body you call 1 and which you call 2.)

Therefore,

$$\frac{1}{R^*} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{0.02} + 0 \Rightarrow R^* = 0.02$$

The maximum pressure (at the center of the contact region is:

$$p_0 = \sqrt{\frac{FE^*}{pR}} = \sqrt{\frac{10^5(1.10x10^{11})}{p\,(0.02)}} = 418 \text{ MPa.}$$

The maximum shear stress is then $\tau_{max} = 0.3\, p_0 = 125$ MPa.