Solutions to Problem Set #9, 24-261

1 (2.4-9)

Reactions at A & B are external loads
Can draw them (when unknown) in
either direction. Here take them as in the
+X direction

one equation
of equil.
2 unknowns
Static. Indet

$\Sigma F_x = R_A + R_D + 5100 - 3400 = 0$

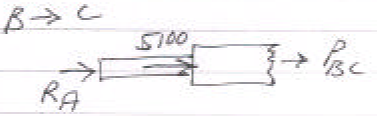
Each of the segments AB, BC and CD will
have a uniform internal load.

Make cut through each section + isolate
to left or right

A → B

$\Sigma F_x = R_A + P_{AB} = 0 \Rightarrow P_{AB} = -R_A$

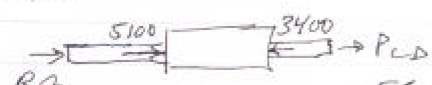
B → C



$\Sigma F_x = R_A + 5100 + P_{BC} = 0$

$P_{BC} = -R_A - 5100$

C → D



$\Sigma F_x = R_A + 5100 - 3400 + P_{CD} = 0$

$P_{CD} = -R_A - 1700$

Note P_{CD} is also $= R_D$

We don't know R_A, but we do know that both ends A & D have $u = 0 \Rightarrow u_A = u_D = 0$

Hence, $\delta_{AB} + \delta_{BC} + \delta_{CD} = (u_B - u_A) + (u_C - u_B) + (u_D - u_C)$

$$= u_D - u_A = 0$$

For each segment $\delta = \text{"}\frac{PL}{EA}\text{"}$

Note: E is same in all segments, but P, L & A are different

$$\delta_{AB} = \frac{P_{AB} L_{AB}}{E A_{AB}} = \frac{(-R_A)(8)}{E(1.2)}$$

$$\delta_{BC} = \frac{P_{BC} L_{BC}}{E A_{BC}} = \frac{(-R_A - 5100)(10)}{E(1.8)}$$

$$\delta_{CD} = \frac{P_{CD} L_{CD}}{E A_{CD}} = \frac{(-R_A - 1700)(8)}{E(1.2)}$$

$\delta_{AB} + \delta_{BC} + \delta_{CD} = 0 \Rightarrow$

$$-8R_A + 10(-R_A - 5100)\left(\frac{1.2}{1.8}\right) + (-R_A - 1700)(8) = 0$$

$$R_A\left[-8 - 10\left(\frac{1.2}{1.8}\right) - 8\right] - 10(5100)\left(\frac{1.2}{1.8}\right) - 8(1700) = 0$$

$$\boxed{\begin{array}{l} R_A = -2100 \text{ lb} \quad (\text{to left}) \\ R_D = -1700 - R_A = 400 \text{ lb} \quad (\text{to right}) \end{array}}$$

(b) Axial force in middle segment is P_{BC}

$$P_{BC} = -R_A - 5100 = 2100 - 5100 = -3000 \text{ lb}$$

$\boxed{BC \text{ is } 3000 \text{ lb in compression}}$

(c) Notice equation governing R_A

$$R_A = -\frac{5100(10)\left(\frac{1.2}{1.8}\right) + 1700(8)}{8 + 10\left(\frac{1.2}{1.8}\right) + 8}$$

Ratio of areas $\frac{1.2}{1.8}$ enters, so changing both by the same factor leave equation the sam

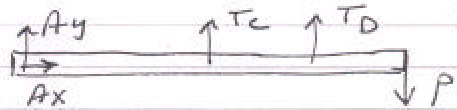
Each term in numerator is a length (10 or 8

" " " Dedominator is a length (10 or 8

so if both lengths 10 & 8 were increased by same factor, equation has same value

$\Rightarrow$ $\boxed{\text{No change in } R_A, R_D \text{ and internal forces}}$

2 (24-14)



P is known

Wires only carry tension

$\Sigma M|_{A_z} = -PL + (T_c)c + (T_D)d = 0$

Note $\Sigma F_x = 0$, $\Sigma F_y = 0$ would bring in A_x, A_y

Need more than statics (3 eqns) to find the four unknowns

Additional relations based on deformation of deformable members.

Wire at C, length h, modulus E, area A

$\delta_c = \dfrac{T_c h}{EA}$

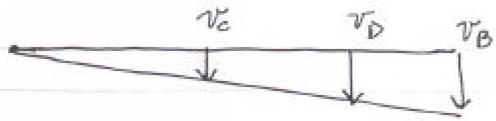
Similar for wire at D, except length is 2h

$\delta_D = \dfrac{2T_D h}{EA}$

Top of both wires is fixed $\Rightarrow$

v_c = downward displacement of point C $= \delta_c$

v_D = " " " " D $= \delta_D$



since bar ACDB is rigid, the displacements are related: $\frac{v_C}{c} = \frac{v_D}{d} = \frac{v_B}{L}$

write $v_D = v_C \frac{d}{c}$

hence $\delta_D = \delta_C \frac{d}{c} \Rightarrow \frac{2T_D h}{EA} = \frac{T_C h}{EA}\frac{d}{c} \Rightarrow T_D = \frac{T_C}{2}\frac{d}{c}$

go back to equilibrium

$$(T_C)c + \frac{T_C}{2}\left(\frac{d}{c}\right)d = PL \Rightarrow T_C = \frac{PL}{c + \frac{d^2}{2c}}$$

$$T_C = \frac{(970)(1.6)}{0.5 + \frac{(1.2)^2}{2(0.5)}} = 800\ N$$

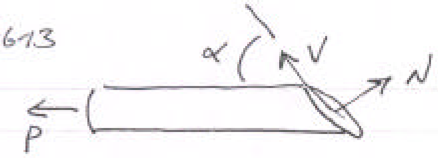
$$T_D = \frac{T_C}{2}\frac{d}{c} = \frac{800}{2}\frac{1.2}{0.5} = 960\ N$$

$$\sigma_C = \frac{T_C}{16\times10^{-6}} = 50\ MPa \qquad \sigma_D = \frac{T_D}{16\times10^{-6}} = 60\ MPa$$

$$v_D = \delta_D = \frac{T_D(2h)}{EA} = \frac{960(2)(0.4)}{(200\times10^9)(16\times10^{-6})} = 0.24\ mm$$

$$v_B = v_D\frac{L}{d} = (0.24)\left(\frac{1.6}{1.2}\right) = \boxed{0.32\ mm = v_B}$$

3. (2.6.13



$\alpha = 36^\circ$

$$\Sigma F_x = -P - V\cos\alpha + N\sin\alpha = 0$$

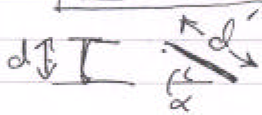
$$\Sigma F_y = V\sin\alpha + N\cos\alpha = 0$$

$$\Rightarrow V = -N\frac{\cos\alpha}{\sin\alpha}$$

$$-V\cos\alpha + N\sin\alpha = P \Rightarrow N\left(\frac{\cos^2\alpha}{\sin\alpha} + \sin\alpha\right) = P$$

$$N = \frac{P\sin\alpha}{\cos^2\alpha + \sin^2\alpha} = P\sin\alpha = 3820 \text{ lb}$$

$$V = -N\frac{\cos\alpha}{\sin\alpha} = -P\cos\alpha = -5260 \text{ lb}$$



$$d'\sin\alpha = d \Rightarrow d' = \frac{d}{\sin\alpha}$$

Area of elliptical surface is 

$$\frac{\pi}{4}(2b)(2a)$$

so area for $d = 2''$ is

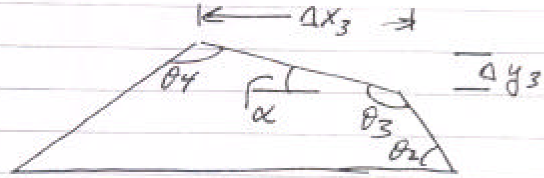
$$\frac{\pi}{4}(d)\left(\frac{d}{\sin\alpha}\right) = \frac{\pi}{4}(2)\frac{2}{\sin 36^\circ} = 5.34 \text{ in}^2$$

shear stress is $\tau = \frac{V}{A} = \frac{5260}{5.34} = \boxed{984 \text{ psi} = \tau}$

normal stress is $\sigma = \frac{N}{A} = \frac{3820}{5.34} = \boxed{715 \text{ psi}}$

Find angles θ_3 and θ_4

(Continuation of Analysis given in problem set)

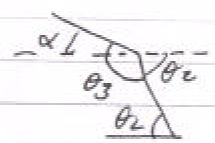


one θ_1, θ_2 are found $\Rightarrow$

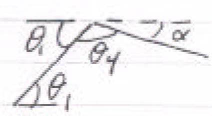
$$\Delta x_3 = L_4 - L_1 \cos\theta_1 - L_2 \cos\theta_2$$

$$\Delta y_3 = L_1 \sin\theta_1 - L_2 \sin\theta_2$$

$$\alpha = \tan^{-1}\left[\frac{\Delta y_3}{\Delta x_3}\right]$$



$$\theta_3 = 180^\circ + \alpha - \theta_2$$



$$\theta_4 = 180^\circ - \alpha - \theta_1$$

can check: $\theta_1 + \theta_2 + \theta_3 + \theta_4$ should $= 360^\circ$

And they do!

Len1	Len2	Len3	Len4
6	3	8	12

Theta 1 (deg)	Theta 1 (rad)	Theta 2 (deg)	Theta 2 (rad)	Calc'd L3	Delta X3	Delta Y3	ThetaBarThre	Theta 3	Theta 4
48	0.837758041	88	1.535889742	8.014748662	7.880517872	1.460696	10.5009055	102.5009	121.4991
48	0.837758041	87	1.518436449	7.963740309	7.828208493	1.46298	10.5856509	103.5857	121.4143
48	0.837758041	89	1.553343034	8.065970875	7.932859143	1.459326	10.4235715	101.4236	121.5764
48	0.837758041	87.5	1.527163095	7.989216521	7.8543582	1.461724	10.5423448	103.0423	121.4577
48	0.837758041	87.6	1.528908425	7.994318554	7.859589401	1.4615	10.5339081	102.9339	121.4661
48	0.837758041	87.7	1.530653754	7.999422805	7.864820984	1.461286	10.5255459	102.8255	121.4745
50	0.872664626	82	1.431169987	7.894898296	7.725755039	1.625462	11.8814691	109.8815	118.1185
50	0.872664626	83	1.448623279	7.944309317	7.777666312	1.618628	11.7561522	108.7562	118.2438
50	0.872664626	84	1.466076572	7.994049882	7.829688952	1.612701	11.6385915	107.6386	118.3614
50	0.872664626	85	1.483529864	8.044098868	7.881807114	1.607683	11.5286921	106.5287	118.4713
50	0.872664626	84.3	1.471312559	8.009033147	7.845315093	1.6111	11.6048216	107.3048	118.3952
50	0.872664626	84.1	1.467821901	7.999041247	7.834896732	1.612158	11.6272584	107.5273	118.3727
54	0.942477796	78	1.361356817	8.080877434	7.849553414	1.919659	13.7423255	115.7423	112.2577
54	0.942477796	77	1.343903524	8.03394814	7.798435323	1.930992	13.9074361	116.9074	112.0926
54	0.942477796	76	1.326450232	7.987502315	7.747522799	1.943215	14.0803224	118.0803	111.9197
54	0.942477796	76.2	1.32994089	7.996751713	7.757688114	1.940699	14.0451205	117.8451	111.9549
54	0.942477796	76.3	1.331686219	8.001383948	7.762774046	1.939455	14.0276369	117.7276	111.9724
60	1.047197551	65	1.134464014	8.119281584	7.732145215	2.477229	17.7644489	132.7644	102.2356
60	1.047197551	64	1.117010721	8.081233378	7.68488656	2.49977	18.0188754	134.0189	101.9811
60	1.047197551	63	1.099557429	8.04398401	7.638028501	2.523133	18.2803644	135.2804	101.7196
60	1.047197551	61	1.064650844	7.971971942	7.545571139	2.572293	18.8242985	137.8243	101.1757
60	1.047197551	62	1.082104136	8.007556055	7.591585312	2.54731	18.5488603	136.5489	101.4511