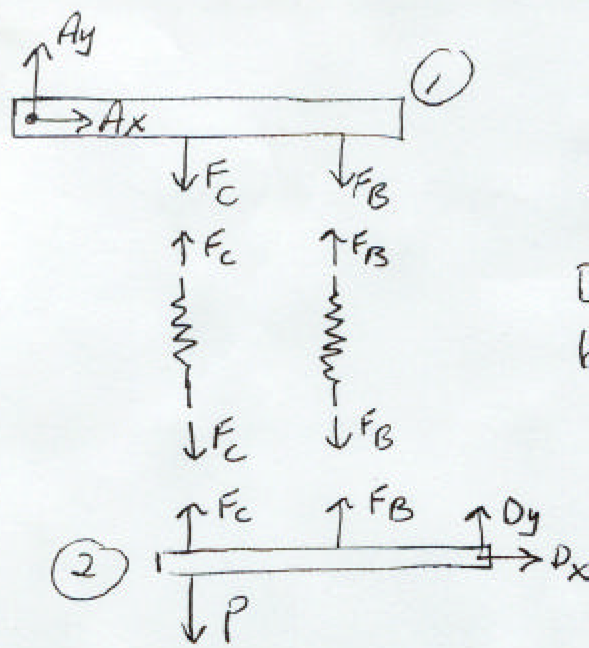
Solutions to Problem Set #8, 24-261, Fall 2001

1 (2.2-7)



Separate bars from springs. Draw forces betw bodies.

Pins at A & D

Body 1 $\quad \Sigma M|_{A_z} = -F_C(b) - F_B(2b) = 0$

$$\Rightarrow F_C = -2F_B$$

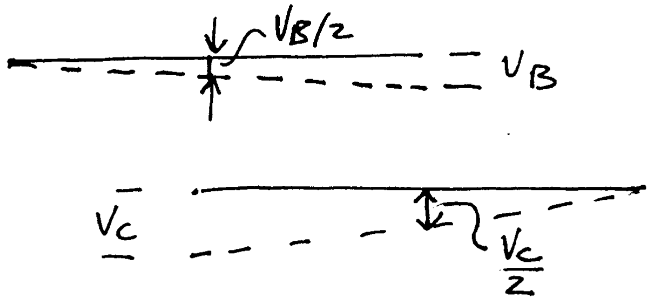
Body 2 $\quad \Sigma M|_{D_z} = P(2b) - F_c(2b) - F_B(b) = 0$

$$\Rightarrow 2P = 2F_c + F_B \Rightarrow 2P = -4F_B + F_B$$

$$\Rightarrow \quad F_B = -2P/3 \Rightarrow F_c = 4P/3$$

$\delta_B = F_B/k = -2P/3k \;;\; \delta_c = F_c/k = 4P/3k$

These are the <u>stretches</u> of ~~the~~ spring A & B.



Let V_c be displacement at point C
" V_B be displacement at point B

Bars are rigid $\Rightarrow$ stay straight when pivoting

By similar triangles, the bar displacements half way toward pivot point is half as much ($V_c/2$ & $V_B/2$)

$$\delta_c = V_c - V_B/2 \qquad\qquad \delta_B = \frac{V_c}{2} - V_B$$

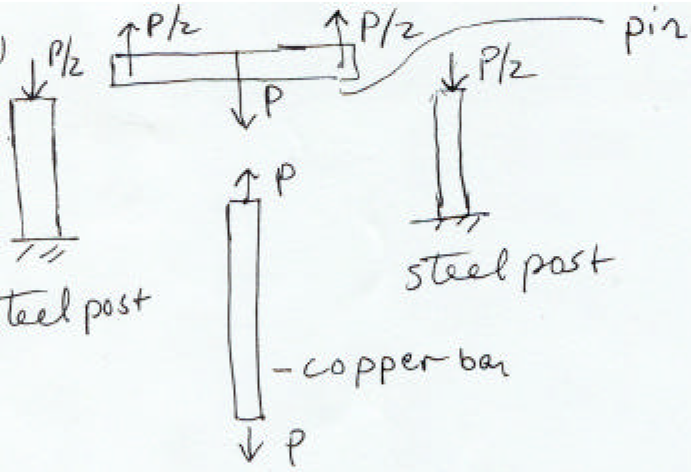
$$\frac{4P_c}{3K} = V_c - V_B/2 \qquad\qquad \frac{-2P}{3K} = \frac{V_c}{2} - V_B$$

$$\Rightarrow V_B = \frac{V_c}{2} + \frac{2P}{3K}$$

$$\frac{4P}{3K} = V_c - \frac{1}{2}\left[\frac{V_c}{2} + \frac{2P}{3K}\right] = \frac{3V_c}{4} - \frac{P}{3K}$$

$$\frac{3V_c}{4} = \frac{5P}{3K} \Rightarrow \boxed{V_c = \frac{20P}{9K}}$$

2 (2.2-11)



Steel posts in compression, copper in tension

In general $\delta = PL/EA$

$$\delta_{steel} = \frac{-\frac{P}{2}(42)}{(30\times10^{6})7} = -10^{-7}P$$

$$\delta_{copper} = \frac{P(25)(12\ \frac{in}{ft})}{(15\times10^{6})(7.5)} = 2.667\times10^{-6}P$$

When steel compresses, pin moves down by $10^{-7}P$

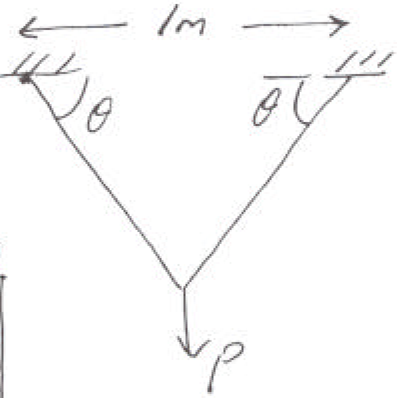
lower end of bar moves down by additional $2.667\times10^{-6}P$

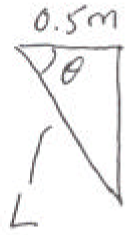
(a) if $P = 90{,}000$ lb $\Rightarrow$ $\delta = (10^{-7} + 2.667\times10^{-6})P = \boxed{0.249'' = \delta}$

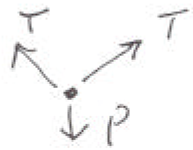
(b) if $\delta = 0.30'' \Rightarrow P = \dfrac{0.3}{10^{-7} + 2.667\times10^{-6}} =$

$P_{max} = 108{,}000$ lb

3 (2.2-14)







$$\Sigma F_y = -P + 2T\sin\theta = 0$$

$$T = \frac{P}{2\sin\theta} = \frac{225\ N}{2\sin 55^\circ} = 137.3\ N$$

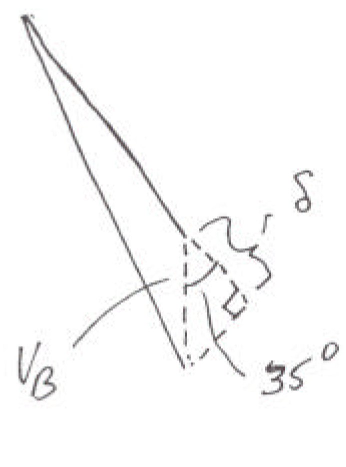
$$\cos\theta = \frac{0.5}{L}$$

$$L = \frac{0.5}{\cos 55^\circ} = 0.872\ m$$

stretch of wire $= \delta = \frac{TL}{EA} = \frac{(137.3)(0.872)}{165,000}$

$$\delta = 0.726 \times 10^{-3}\ m = 0.726\ mm$$

Find motion down of point B

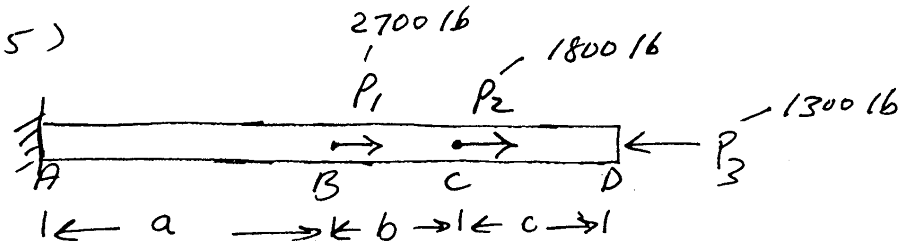


$$\cos 35^\circ = \frac{\delta}{V_B}$$

$$V_B = \frac{\delta}{\cos 35^\circ} = \frac{0.726}{\cos 35^\circ}$$

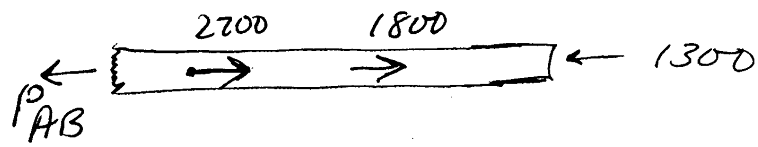
$$V_B = 0.886\ m$$

4 (2.3-5)



Find internal forces in segments

AB

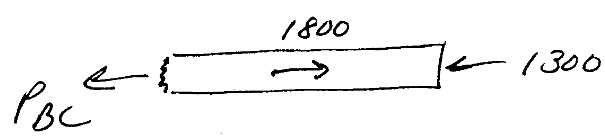


P_{AB} drawn in tension

$$\Sigma F_x = -P_{AB} + 2700 + 1800 - 1300 = 0$$

$$P_{AB} = 3200 \text{ lb}$$

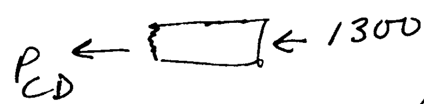
BC



$$\Sigma F_x = -P_{BC} + 1800 - 1300 = 0$$

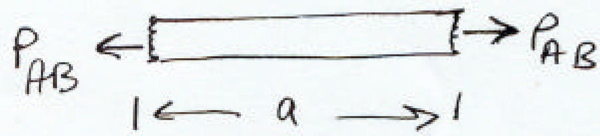
$$P_{BC} = 1800 - 1300 = 500 \text{ lb}$$

CD



$$\Sigma F_x = -P_{CD} - 1300 = 0 \Rightarrow P_{CD} = -1300 \text{ lb}$$

Each segment is like a bar in uniaxial tension



$$\delta_{AB} = \frac{P_{AB} L_{AB}}{E\ A} = \frac{(3200)(\overset{60''}{a})}{(30 \times 10^6)(0.4)} = 1.6 \times 10^{-2} \text{ in}$$

$$\delta_{BC} = \frac{P_{BC}\, b}{E A} = \frac{(500)(24)}{(30 \times 10^6)(0.4)} = 1 \times 10^{-3} \text{ in}$$

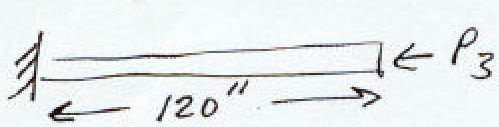
$$\delta_{CD} = \frac{P_{CD}\, c}{E A} = \frac{(-1300)(36)}{(30 \times 10^6)(0.4)} = -3.9 \times 10^{-3}$$

pieces are connected $\Rightarrow$ total stretch is sum of the stretches

$$\delta = \delta_{AB} + \delta_{BC} + \delta_{CD} = 1.31 \times 10^{-2}$$

Since left end is fixed $\Rightarrow$ moves to right

Add axial compressive force at end D: P_3'

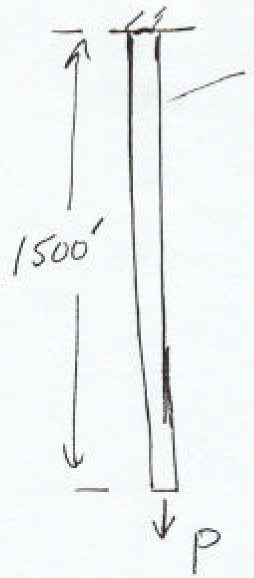


$$\delta_{AD} = \frac{-P_3'(120)}{(30 \times 10^6)(0.4)}$$

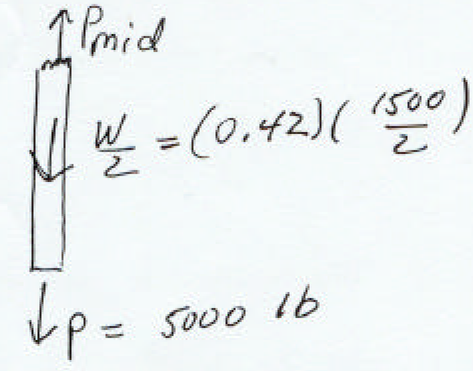
want $\delta_{AD} = -1.31 \times 10^{-2} \Rightarrow P_3' = 1310$ lb

P_3 should be increased by 1310 lb

5 (2.3-9)



Internal load at mid-section P_{mid}



$$P_{mid} = 0.42\ \frac{1500}{2} + 5000 = \quad 5315\ \text{lb}$$

$$\sigma = \frac{P_{mid}}{A} \qquad \varepsilon_{mid} = \frac{\sigma}{E} = \frac{P_{mid}}{EA} = 0.00235$$

$$\text{Find } EA = \frac{P_{mid}}{0.00235} = \frac{5315}{.00235} = 2.26\times10^6\ \text{lb}$$

$$\text{At upper end: } P_{upper} = (0.42)(1500) + 5000$$
$$= 5630$$

$$\varepsilon_{upper} = \frac{P_{upper}}{EA} = \frac{5630}{2.26\times10^6} = 0.00249$$

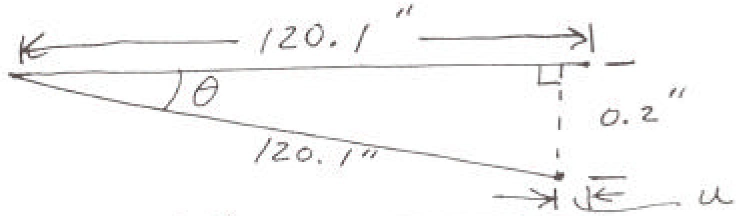
$$\varepsilon_{lower} = \frac{5000}{EA} = 0.00221$$

$$\delta = \int_0^L \varepsilon\, dz = \int_0^{1500}\left[.00249 - .00028\left(\frac{z}{1500}\right)\right]dz$$

$$\delta = \underbrace{(.00249)(1500) - .00028\,\frac{z^2}{2(1500)}\Big|_0^{1500}}_{\text{same as } (\varepsilon_{mid})(1500')}$$

$$\delta = 3.525\ \text{ft} = 42.3''$$

6.



$\sin\theta = \frac{0.2}{120.1} \Rightarrow \boxed{\theta = 0.0954^\circ}$

horizontal length is

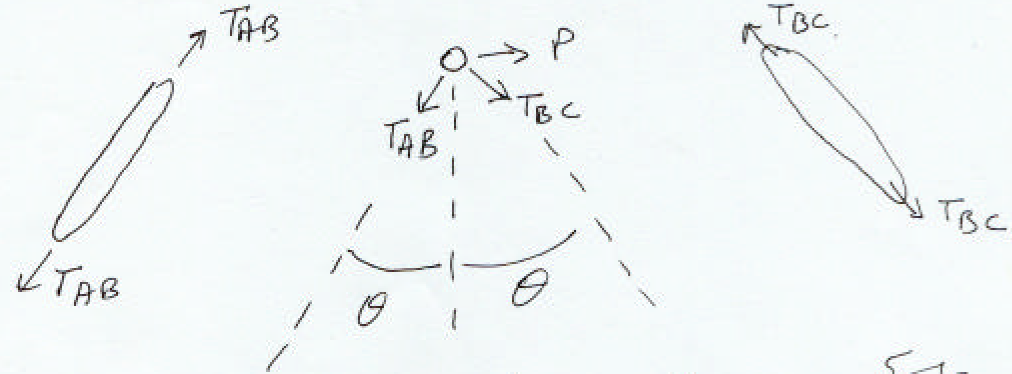
$(120.1)\cos\theta \Rightarrow u = 120.1 - (120.1)\cos\theta$

$= 1.665 \times 10^{-4}$ in

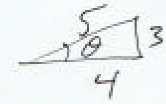
hence position of right end is

$0.1 - 1.665\times10^{-4} = \boxed{0.09983''}$ to the right of the initial position

7(2.7-6) Each is a two-force member



$$\tan\theta = \frac{L/2}{H} = \frac{L/2}{2L/3} = \frac{3}{4}$$



Equilibrium of pin

$$\Sigma F_y = -T_{AB}\cos\theta - T_{BC}\cos\theta = 0 \Rightarrow T_{AB} = -T_{BC}$$

$$\Sigma F_x = -T_{AB}\sin\theta + T_{BC}\sin\theta + P = 0$$

$$\Rightarrow 2T_{BC}\sin\theta + P = 0 \Rightarrow T_{BC} = -T_{AB} = -\frac{P}{2\sin\theta}$$

strain energy of truss = Σ(strain energy)

Bar AB : $U_{AB} = \dfrac{T_{AB}^2 (\text{length})}{2EA}$

$$\text{length} = \sqrt{(L/2)^2 + H^2} = \sqrt{(L/2)^2 + 4L^2/9} = L\sqrt{\frac{9+16}{36}} = \sqrt{\frac{25}{36}}\,L$$

$$U_{AB} = \frac{\left[\frac{P}{2(3/5)}\right]^2 \sqrt{\frac{25}{36}}\,L}{2EA} = \frac{\frac{25}{36}\sqrt{\frac{25}{36}}\,L\,P^2}{2EA}$$

$$= \frac{125}{216}\,\frac{P^2 L}{2EA}$$

Since $T_{BC} = -T_{AB}$, $U_{BC} = U_{AB}$

$U =$ total strain energy of truss $= \frac{125}{216} \frac{P^2 L}{EA}$

Let u_B be horizontal displacement of B

$$\frac{1}{2} P u_B = U$$

$$\Rightarrow u_B = \frac{2U}{P} = \frac{125}{108} \frac{PL}{EA}$$

^{8}(2.8-12)

mass of 50 kg will fall a distance $L + \delta$ where L is unstretched length of cord + δ is the stretch.

Hence decrease in potential energy is

$$(50)(9.81)(L+\delta)$$

Given that total length of fall is

$$60 - 10 = 50 \text{ m}$$

$$L + \delta = 50 \text{ m}$$

Energy is cord under a stretch δ is

$$\frac{1}{2} P\delta = \frac{1}{2}\left(\frac{EA\delta}{L}\right)\delta = \frac{EA\delta^2}{2L} = \frac{2100}{2}\frac{\delta^2}{L}$$

$$50(9.81)(50) = \frac{2100}{2}\frac{\delta^2}{L} = \frac{2100}{2}\frac{(50-L)^2}{L}$$

$$23.36\, L = (50-L)^2 = 2500 - 100L + L^2$$

$$L^2 - 123.36L + 2500 = 0 \Rightarrow L = \frac{123.36 \pm \sqrt{(123.36)^2 - 4(250}}{2}$$

$$L = \frac{123.36 \pm 72.23}{2}$$ must take "−" root

$\boxed{L = 25.6 \text{ m}}$ substitute back to test