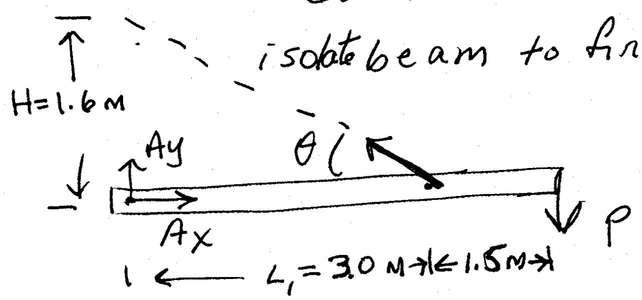
Solutions to Set #7, 24-261, Fall 2001

1. (1.2-10) cable takes a tensile force T

isolate beam to find T

A is a pin joint



$$\Sigma M|_{A_z} = -P(4.5) + T\sin\theta\,(3) = 0$$

$$\theta = \tan^{-1}\left(\frac{1.6}{3}\right) = 28.1^\circ$$

$$T = \frac{P(4.5)}{3\sin\theta} = 3.19\,P$$

if $P = 32$ kN $\Rightarrow$ $T = 102$ kN

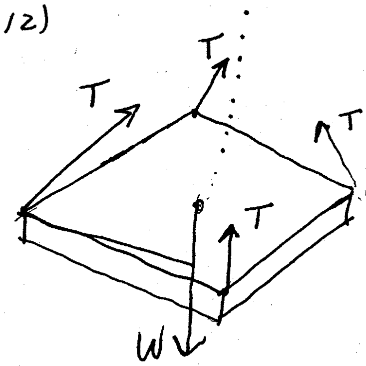
cable has area of $481\ mm^2 = 481\times10^{-6}\ m^2$

stress in cable $\sigma = \dfrac{102{,}000}{481\times10^{-6}} = 212\times10^{6}$ Pa $\boxed{\sigma = 212\ \text{MPa}}$

cable length $= \sqrt{1.6^2 + 3^2} = 3.4$ m

strain in cable $\varepsilon = \dfrac{5.1\times10^{-3}}{3.4} = \boxed{0.0015 = \varepsilon}$

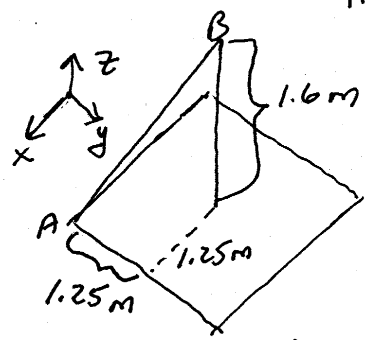
2. (1.2-12)



All cables run through a common point. By symmetry they have the same tension.

The slab weight is $\rho g V = \gamma V$

$$W = 24 \frac{kN}{m^3}[2.5][2.5][.225] = 33.75\ kN$$



will be taking $\Sigma F_z = 0$ so want z-component of cable force

$$|\underset{\sim}{AB}| = \sqrt{(1.25)^2 + (1.25)^2 + (1.6)^2} = 2.384$$

force in z-direction is $T\left[\frac{1.6}{2.384}\right] = 0.671 \cdot$

$$\Sigma F_z = 4[0.671\ T] - 33.75 = 0 \Rightarrow T = 12.6\ kN$$

Area $A = 190\ mm^2 = 190 \times 10^{-6}\ m^2$

$$\sigma = \frac{12{,}600}{190 \times 10^{-6}} = 66.2 \times 10^6\ Pa = 66.2\ MPa$$

3 (1.3-7

	Diameter = 0.505 in		
	Gage Length = 2.0 in		
Load (lb)	Elongation (in)	strain	stress (psi)
1000	0.0002	0.0001	4992.606783
2000	0.0006	0.0003	9985.213565
6000	0.0019	0.00095	29955.6407
10000	0.0033	0.00165	49926.06783
12000	0.0039	0.00195	59911.28139
12900	0.0043	0.00215	64404.6275
13400	0.0047	0.00235	66900.93089
13600	0.0054	0.0027	67899.45224
13800	0.0063	0.00315	68897.9736
14000	0.009	0.0045	69896.49496
14400	0.0102	0.0051	71893.53767
15200	0.013	0.0065	75887.6231
16800	0.023	0.0115	83875.79395
18400	0.0336	0.0168	91863.9648
20000	0.0507	0.02535	99852.13565
22400	0.1108	0.0554	111834.3919
22600			112832.9133

From plot, can see that σ vs. ϵ is linear from low loads up to ≈ 12900 lb

Find slope of line using change in stress over change in strain

$$E \approx \frac{64405 - 9985}{.00215 - .0003} = 29.4 \times 10^6 \text{ psi}$$

Find proportional limit from where curve deviates from linear

use a millimeter scale on graph

$$\sigma_{P.L.} = 60{,}000 + 20{,}000\left[\frac{6}{18}\right] \approx 66{,}700 \text{ psi}$$

↑ mm on graph

$$\boxed{\sigma_{P.L.} = 66{,}700 \text{ psi}}$$

Draw line parallel to initial linear pars

$$\sigma_{0.1\%\ offset} = 60{,}000 + 20{,}000\left[\frac{8}{18}\right] = 68{,}900 \text{ ps}$$

$$\boxed{\sigma_{0.1\%\ offset} = 68{,}900 \text{ psi}}$$

Final elongation between gage marks is 0.12

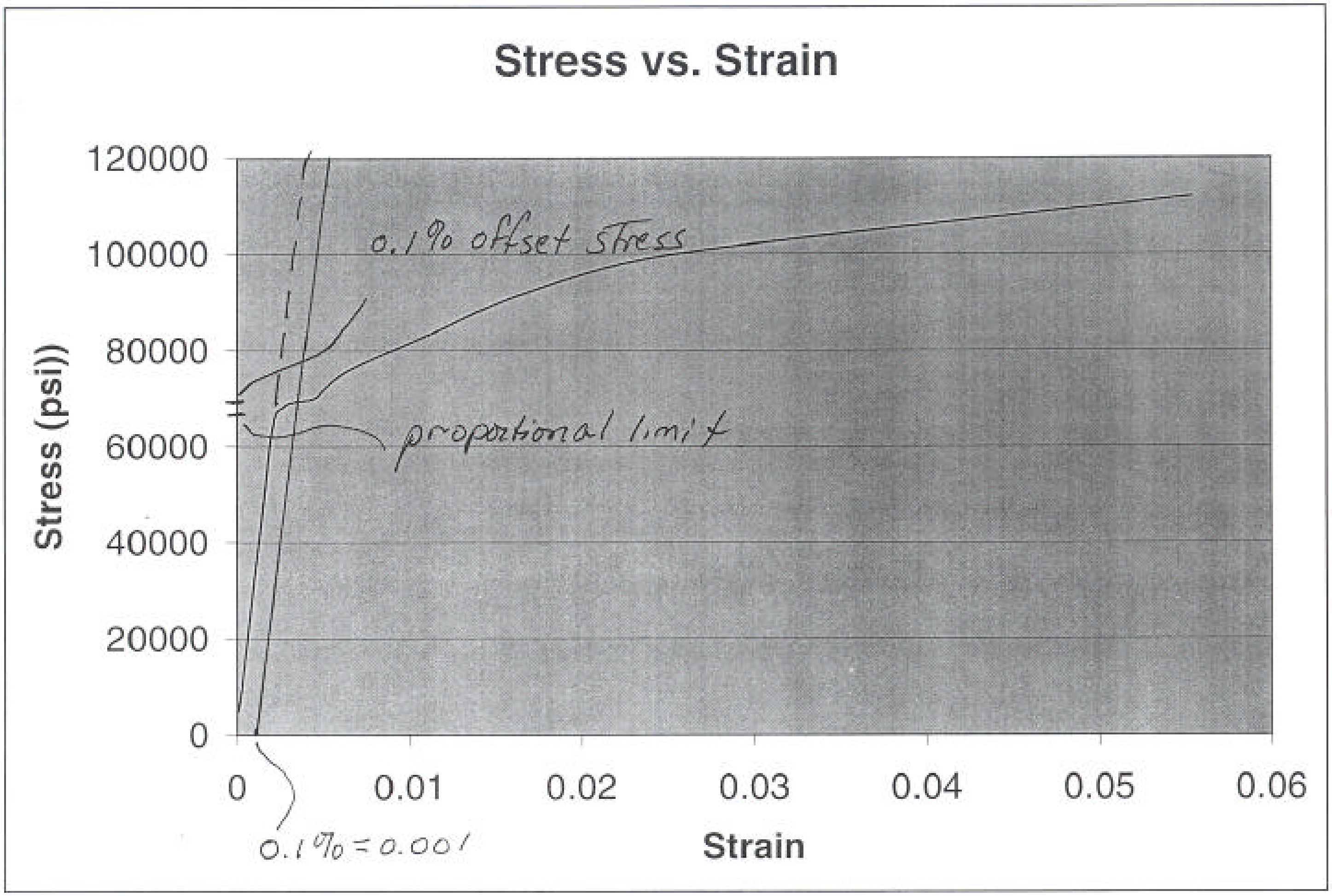
$$\frac{0.12}{2} = 0.06 = 6\%$$

Final diameter = $0.42'' \Rightarrow Area_f = 0.139 \text{ in}^2$

Initial area = $\frac{\pi}{4}(.505)^2 = 0.200 \text{ in}^2$

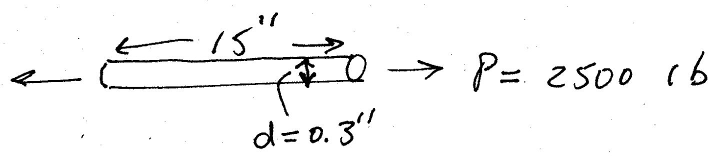
$$\% \text{ reduction in area} = 100\left[\frac{0.200 - 0.139}{0.200}\right]$$

$$\boxed{\text{Area Reduction} = 30.7\%}$$



Stress vs. Strain
Stress (psi))
120000
100000
80000
60000
40000
20000
0
0
0.01
0.02
0.03
0.04
0.05
0.06
Strain
0.1% offset stress
proportional limit
0.1% = 0.001

4 (1.5-5)



$$\sigma = \frac{P}{A} = \frac{2500}{\frac{\pi}{4}(0.3)^2} = 35,400 \text{ psi}$$

$$\varepsilon = \frac{\sigma}{E} = \frac{35,400}{25 \times 10^6} = 1.41 \times 10^{-3}$$

$$\delta = L\varepsilon = (15)(1.41 \times 10^{-3}) = \boxed{0.0212'' = \delta}$$

transverse strain $\varepsilon_t = -\nu\varepsilon$

$$\varepsilon_t = -0.32\,(1.41 \times 10^{-3}) = -4.53 \times 10^{-4}$$

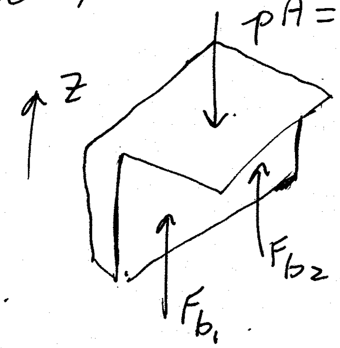
$$\text{Final diameter} = 0.3 + (-4.53 \times 10^{-4})(0.3)$$
$$= 0.29986'' = d_f$$

$$\text{Area (final)} = \frac{\pi}{4} d_f^2 = 7.06218 \times 10^{-2}$$
$$\text{Area (initial)} = \frac{\pi}{4}(.3)^2 = 7.06858 \times 10^{-2}$$

$$\% \text{ change} = (100)\,\frac{7.06218 \times 10^{-2} - 7.06858 \times 10^{-2}}{7.06858 \times 10^{-2}}$$

$$\boxed{.091\% \text{ decrease in area}}$$

5. (1.6-2)



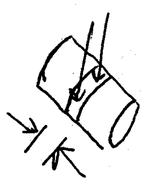
$pA = (2 \times 10^6 \frac{N}{m^2})(0.15)(0.06)$
$= 1.8 \times 10^4 \; N$

By symmetry or sum of moments $F_{b_1} = F_{b2}$

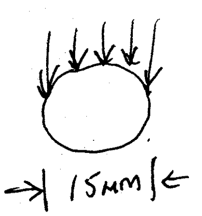
$\Sigma F_z = -1.8 \times 10^4 + 2 F_{b_1} = 0$

$F_{b_1} = F_{b2} = 9000 \; N$

Bearing pressure (average) flange on bolt



or

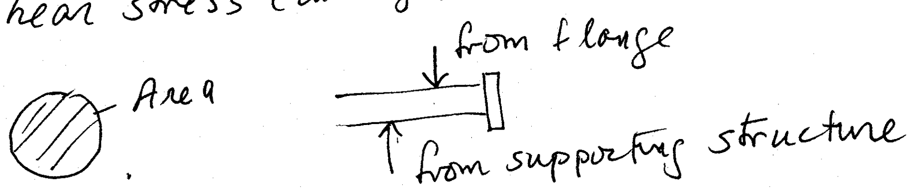


$t = 12 \, mm$

Bearing area is
$(12)(15) =$
$180 \, mm^2$
$= 180 \times 10^{-6} \, m^2$

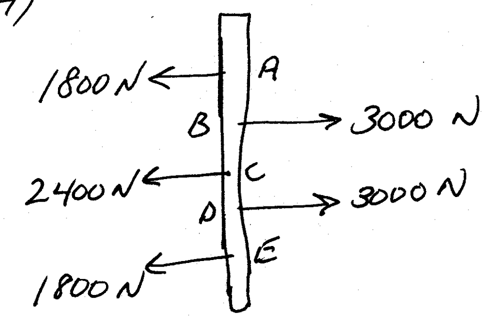
$$\sigma_{bearing} = \frac{9000}{180 \times 10^{-6}} = 50 \times 10^6 \; Pa = 50.0 \, MPa$$

Shear stress (average)



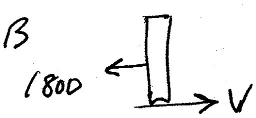
$$\tau = \frac{9000}{\frac{\pi}{4}(.015)^2} = 50.9 \; MPa$$

6 (1.6-4)



Shear stress has a different value in four region

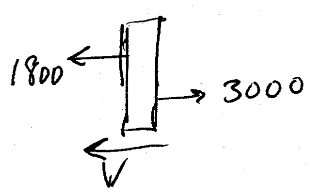
A → B



V = 1800 N

same for D → E

B → C



V = 1200 N

same for C → D

max shear stress is $\dfrac{1800}{\frac{\pi}{4}(.006)^2} = 63.7$ MPa

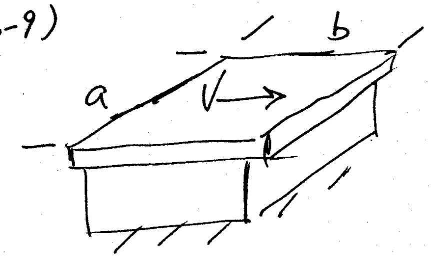
bolt diameter

Bearing forces relate to forces on plates. Largest plate force is 3000 N

Bearing area = (plate thickness)(bolt diameter)

$\sigma_{bearing} = \dfrac{3000}{(.005)(.006)} = 100$ MPa

7. (1.6-9)



shear stress is

$$\tau = \frac{V}{(a)(b)} = \frac{1200}{(5)(6)}$$

$$\tau = 40 \text{ psi}$$

shear strain is $\frac{\text{displacement}}{\text{thickness}} = \frac{0.24}{1.5} = 0.16$

shear modulus G relates τ & γ

$$\tau = G\gamma$$

$$\Rightarrow G = \frac{\tau}{\gamma} = \frac{40}{0.16} = 250 \text{ psi}$$