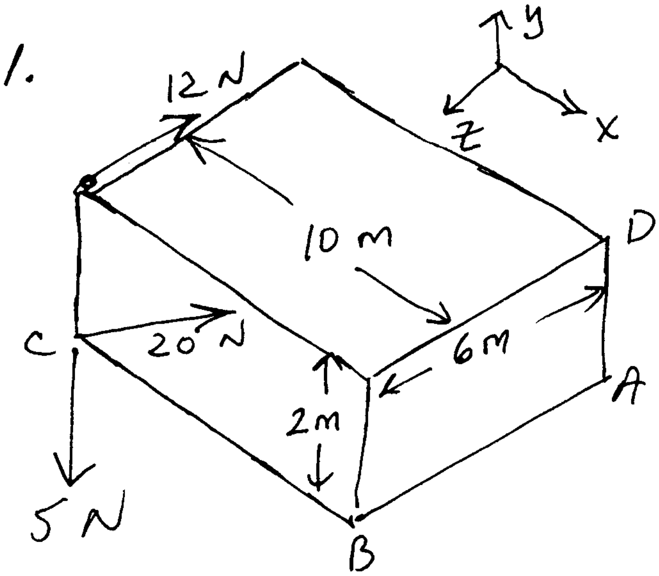
Solutions to Set # 2, 24-261, Fall 2001

1.



Consider 5 N force + its moment about D

- about x-axis
  $d = 6$ m
  $M|_{D_x} = 5(6) = 30$ N-m
- about y-axis - no moment
- about z-axis, $d = 10 \Rightarrow M|_{D_z} = 5(10) = 50$ N-m

Consider 12 N force + moment about A

- about x-axis, $d = 2$ m
  $M|_{A_x} = -12(2) = -24$ N-m (rotation is about $-x$ by right hand rule)
- about y-axis, $d = 10$ m
  $M|_{A_y} = -12(10) = -120$ N-m
- about z-axis - no moment

Consider 20 N force + moment about A

First resolve force into components

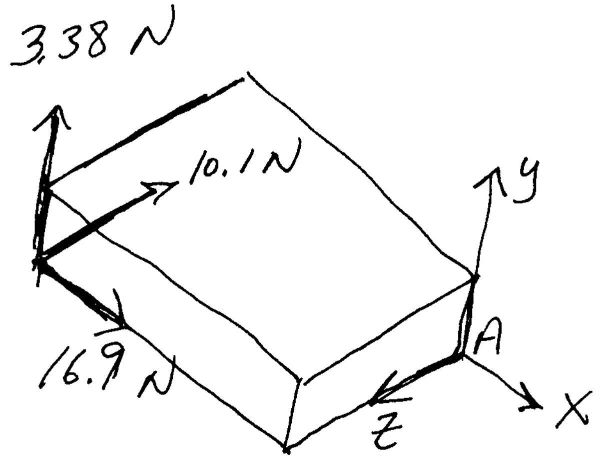
Vector from C to D is

$$\underset{\sim}{CD} = 10\,\underset{\sim}{i} + 2\,\underset{\sim}{j} - 6\,\underset{\sim}{k}$$

$$|\underset{\sim}{CD}| = \sqrt{10^2 + 2^2 + 6^2} = 11.83$$

unit vector $\underset{\sim}{u} = \dfrac{\underset{\sim}{CD}}{|\underset{\sim}{CD}|} = 0.845\,\underset{\sim}{i} + 0.169\,\underset{\sim}{j} - 0.507\,\underset{\sim}{k}$

$$\underset{\sim}{F}_{CD} = 20\,\underset{\sim}{u} = 16.9\,\underset{\sim}{i} + 3.38\,\underset{\sim}{j} - 10.1\,\underset{\sim}{k} \text{ N}$$



Consider moment about each axis at a time

x-axis : only 3.38 N in y-direction has a moment

$d = 6$ m ; moment is about $-x$ axis

$$M|_{A_x} = -3.38(6) = -20.3 \text{ N-m}$$

<u>y - axis</u> : both x & z components produce a moment about y

16.1 N in x : d = 6, moment about +y

10.1 N in -z : d = 10, moment about -y

$M/_{A_y} = 16.9(6) - 10.1(10) = 0.4$ N-m

Actually, the moment should be zero.
The Force $\underset{\sim}{F}_{CO}$ runs through the axis since the y-axis through A runs through O
The 0.4 N-m is due to round off

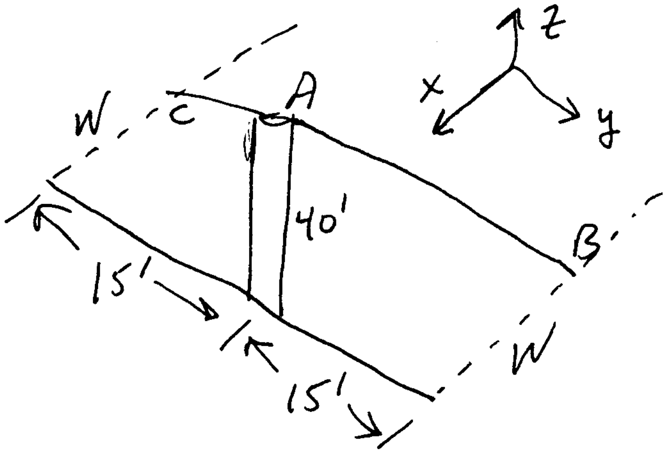
<u>z - axis</u> : only 3.38 N in y-direction has a moment

d = 10 m ; moment is about -z axis

$M/_{A_z} = -3.38(10) = -33.8$ N-m

$M/_{A_x} = -20.3$ N-m , $M/_{A_y} = 0$, $M/_{A_z} = -33.8$ N-m

2.



say cables are attached a distance W behind the pole

cable tensions are 200 lb each

Write down unit vectors for $\underset{\sim}{AB}$ and $\underset{\sim}{AC}$ and then multiply by 200 lb to get forces $\underset{\sim}{F}_{AB}$ and $\underset{\sim}{F}_{AC}$

$$\underset{\sim}{F}_{AB} = 200\left[\frac{-W\underset{\sim}{i} + 15\underset{\sim}{j} - 40\underset{\sim}{k}}{\sqrt{W^2+15^2+40^2}}\right]$$

$$\underset{\sim}{F}_{AC} = 200\left[\frac{-W\underset{\sim}{i} - 15\underset{\sim}{j} - 40\underset{\sim}{k}}{\sqrt{W^2+15^2+40^2}}\right]$$

Note that $\underset{\sim}{k}$-component is parallel to pole

the $\underset{\sim}{j}$-components from $\underset{\sim}{F}_{AB}$ & $\underset{\sim}{F}_{AC}$ cancel

Thus, only $\underset{\sim}{i}$-components add

Net force perpendicular to pole is

$$\frac{-(2)(200)W}{\sqrt{W^2+15^2+40^2}}\ \underset{\sim}{i}\ \text{lb}$$

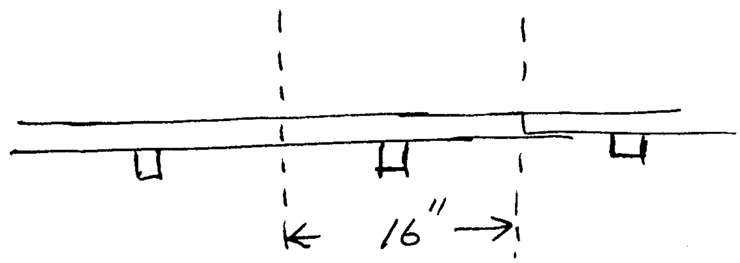
Net moment about the base is due to this $\underset{\sim}{i}$ -component. Moment arm is 40 ft. The moment is about the $-y$ axis.

Net moment is

$$\frac{-(2)(200)(40)\,W}{\sqrt{W^2+15^2+40^2}}\,\underset{\sim}{j} \quad \text{lb-ft}$$

Distance (ft)	Force (lb) in -x direction	Moment (lb-ft) about -y axis
5	46.4990555	1859.96222
6	55.6337142	2225.348568
7	64.68048058	2587.219223
8	73.62647348	2945.058939
9	82.4595675	3298.3827
10	91.16846117	3646.738447
11	99.74273176	3989.70927
12	108.1728767	4326.915068
13	116.4503415	4658.013662
14	124.5675349	4982.701397
15	132.5178313	5300.713252
16	140.2955622	5611.82249
17	147.8959976	5915.839905
18	155.3153175	6212.612698
19	162.5505763	6502.023051
20	169.5996608	6783.986432
21	176.4612427	7058.449707
22	183.1347271	7325.389085
23	189.620199	7584.807961
24	195.9183673	7836.734694
25	202.0305089	8081.220356

3.



Books are 9.5" high and 8.25" deep (into paper)

consider a 16" stretch of shelf spanning one bracket. This contains

$\frac{16}{1.6} = 10$ books or $(10)(4) = 40$ lb

These books lie on $(16)(8.25) = 132 \text{ in}^2$

Force per area of plank $= \frac{40}{132} = \boxed{0.303 \frac{\text{lb}}{\text{in}^2}}$

The 40 lb of books lie on 16" of plank, so

Force per length of plank $= \frac{40}{16} = \boxed{2.5 \frac{\text{lb}}{\text{in}}}$

The volume of plank is $(8.25)(16)(0.75) = 99 \text{ in}^3$

convert volume to $\frac{99}{(12)^3} = 0.053 \text{ ft}^3$

weight of plank $= (40 \frac{\text{lb}}{\text{ft}^3})(0.053 \text{ ft}^3) = 2.29$ lb

Bracket takes $40 + 2.29 = \boxed{42.29 \text{ lb}}$

top of bracket: area $= (0.125)(8.25) = 1.03 \text{ in}^2$

Force per area on bracket $= \frac{42.29}{1.03} = 41.0 \frac{\text{lb}}{\text{in}^2}$

Bracket length = 8.25"

Force per length of bracket $= \frac{42.28}{8.25} = 5.13 \frac{\text{lb}}{\text{in}}$