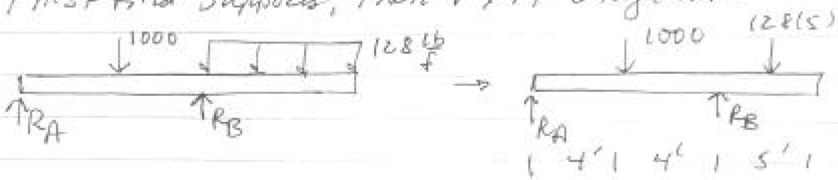
Solution to Set #14, 24-261, Fall 2001

1 (5.5-21)

First find supports, then V & M diagrams



$$\Sigma M_A = -1000(4) + R_B(8) - 128(5)\left(\tfrac{5}{2}+8\right) = 0$$

$$R_B = \frac{4000 + 5(128)(10.5)}{8} = 1340 \text{ lb}$$

$$\Sigma F_y = R_A + R_B - 1000 - 5(128) = 0 \Rightarrow R_A = 300 \text{ lb}$$

$300(4) = 1200$

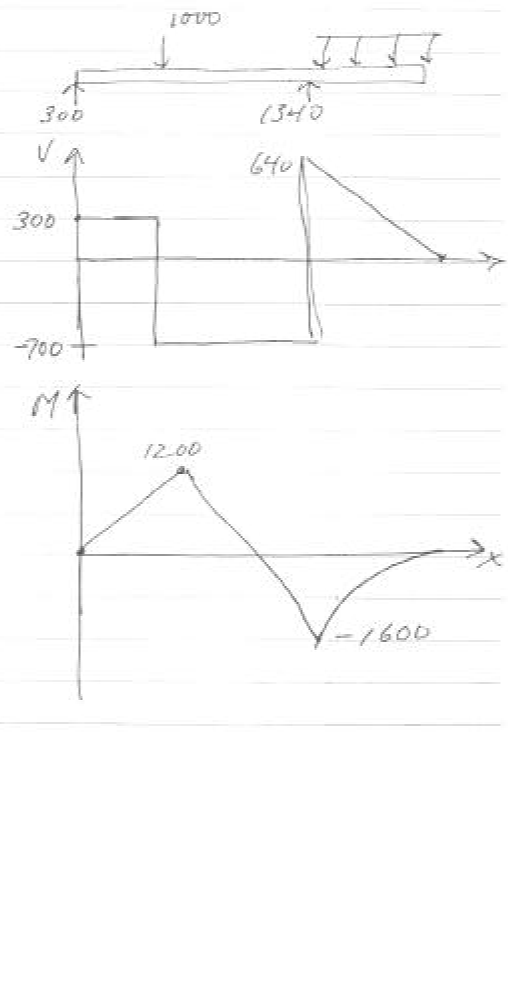
$1200 - 4(700) = -1600$

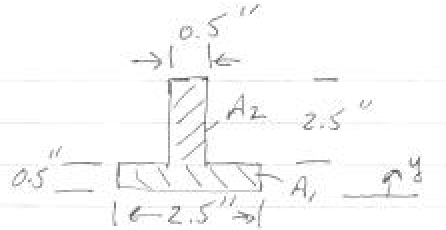
$M_{max} = 1200$ lb-ft

$M_{min} = -1600$ lb-ft

or $M_{max} = 14{,}400$ lb-in

$M_{min} = -19{,}200$ lb-in





$A_1 = (2.5)(0.5) \quad \bar{y}_1 = 0.25$

$A_2 = (2.5)(0.5) \quad \bar{y}_2 = 0.5 + 1.25$

$$\bar{y} = \frac{1}{A_1 + A_2}\left[A_1\bar{y}_1 + A_2\bar{y}_2\right] = \frac{1}{2.5}\left[(1.25)(0.25) + (1.25)(1.75)\right]$$

$$= 1'' = \bar{y}$$

Find $I = \int y^2 dA$ where y is measured from centroid $\bar{y}$

$$I = \int_{A_1} y^2 dA + \int_{A_2} y^2 dA = I_1 + I_2$$

$I_1 = I_{c_1} + A_1 d^2$ parallel axis th^m

$$I_1 = \frac{1}{12}(2.5)(0.5)^3 + (1.25)\left[1 - 0.25\right]^2$$

$$I_2 = \frac{1}{12}(0.5)(2.5)^3 + (1.25)\left[1.75 - 1\right]^2$$

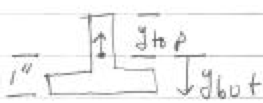
$I_1 + I_2 = 2.083$

At $M_{max} = 14{,}400$, $\sigma_{top} = \dfrac{-M y_{top}}{I} = \dfrac{-14{,}400(2)}{2.083} = -13{,}800$ psi

$\sigma_{bottom} = \dfrac{-M y_{bot}}{I} = \dfrac{-14{,}400(-1)}{2.083} = 6900$ psi

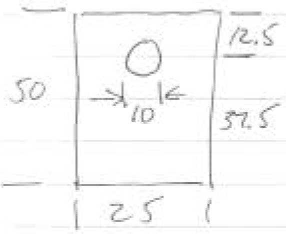
At $M_{min} = -19200$, $\sigma_{top} = \dfrac{-M y_{top}}{I} = \dfrac{-(-19200)(2)}{2.083} = 18{,}400$ psi

$\sigma_{bot} = \dfrac{-M y_{bot}}{I} = \dfrac{-(-19200)(-1)}{2.083} = -9200$ psi



$\boxed{\sigma_{tension} = 18{,}400 \text{ psi}, \ \sigma_{comp} = -13800 \text{ psi}}$

2(5.5-22) Find cross-sectional properties $\bar{y}$, I



half a solid rectangle
subtract off the hole
(All dimensions are in mm)

A_1 is $(25)\times(50)$ rectangle
A_2 is $\frac{\pi}{4}d^2$ circle

$\bar{y}_1 = 25$, $\bar{y}_2 = 37.5$

since $\bar{y} = \frac{1}{A}\int y\,dA = \frac{1}{A}\left[\int_{A_1} y\,dA - \int_{A_2} y\,dA\right]$

$$\bar{y} = \frac{1}{A_1 - A_2}\left[A_1\bar{y}_1 - A_2\bar{y}_2\right]$$

$$= \frac{1}{1250 - 78.5}\left[(1250)(25) - (78.5)(37.5)\right] = \boxed{24.16\ \text{mm} = \bar{y}}$$

$I = \int y^2\,dA$ where y is measured from $\bar{y}$

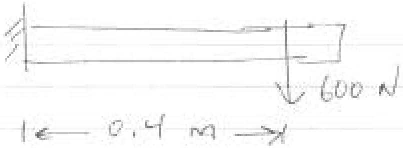
$$I = \int_{A_1} y^2\,dA - \int_{A_2} y^2\,dA = I_1 - I_2$$

use parallel axis thm

$$I_1 = \frac{1}{12}(25)(50)^3 + (25)(50)\left[25 - 24.16\right]^2$$

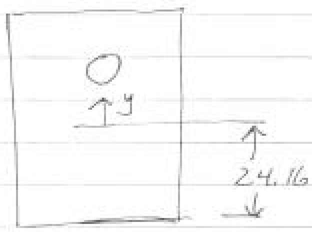
$$I_2 = \frac{\pi}{4}(5)^4 + \frac{\pi}{4}(10)^2\left[37.5 - 24.16\right]^2$$

$$I_1 - I_2 = 2.468\times10^5\ \text{mm}^4 = 2.468\times10^{-7}\ \text{m}^4$$



Maximum moment is at support, where it is

$M = -(0.4)(600) = -240$ N-m



top $y = 50 - 24.16 = 25.84$ mm

$\sigma = \frac{-My}{I} = \frac{240\,(.02584)}{2.468 \times 10^{-7}} = 25.1$ mPa

top of hole
$y = 37.5 - 24.16 + 5 = 18.34$ mm

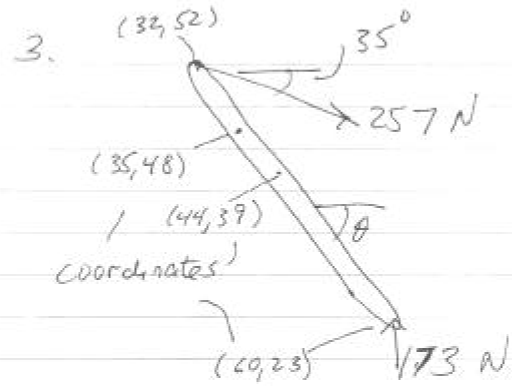
$\sigma = \frac{-My}{I} = \frac{240\,(.01834)}{2.468 \times 10^{-7}} = 17.8$ mPa

bottom $y = -24.16$

$\sigma = \frac{-My}{I} = \frac{(240)(-.02416)}{2.468 \times 10^{-7}} = -23.5$ mPa

$\boxed{\sigma_{top} = 25.1 \text{ mPa}, \quad \sigma_{top\ hole} = 17.8 \text{ mPa}, \quad \sigma_{bottom} = -23.5 \text{ mPa}}$

3.

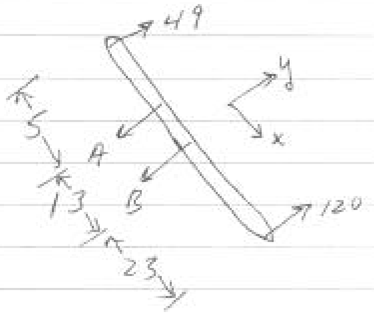


$$\tan\theta = \frac{52-23}{60-32} \Rightarrow \theta = 46^\circ$$

perpendicular forces are

$$257 \sin(46-35) = 49$$

$$173 \cos(46) = 120$$

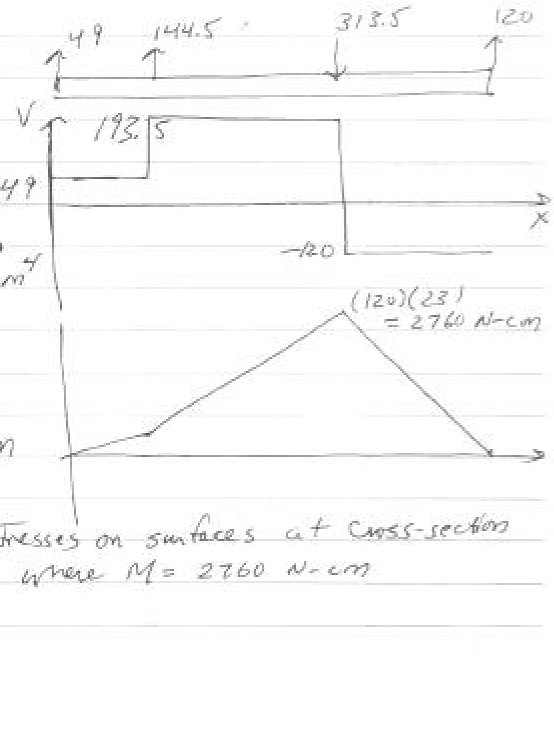


$$\Sigma M|_{A_z} = -49(5) - B(13) + 120(36) = 0$$

$$B = 313.5$$

$$\Sigma F_y = 49 - A - 313.5 + 120 = 0$$

$$A = -144.5$$



$$M_{max} = 27.60 \text{ N-m}$$

$$I = \frac{\pi}{4}(r_o^4 - r_i^4)$$

$$= \frac{\pi}{4}(1^4 - (0.8)^4)$$

$$= 0.464 \text{ cm}^4 = 4.64 \times 10^{-9} \text{ m}^4$$

$$\sigma = \frac{My}{I} = \frac{(27.6)(.01)}{4.64 \times 10^{-9}}$$

$$\boxed{\sigma = 59.5 \text{ MPa}}$$

max stresses on surfaces at cross-section where M = 2760 N-cm