Solutions to Prob Set #13, 24-261, Fall 2001

1. (5.4-4)

Beam is under pure bending (two equal + opposite moments M_0)

The beam deforms into an arc of a circle with radius ρ.

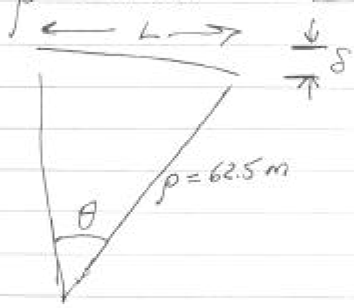
The strain at the top is 0.0008, and the distance from the neutral axis to the top surface is 50 mm

where $\varepsilon = 0$

Define y from neutral surface. Since bending is such that strain is tensile on top

$$\varepsilon = \frac{y}{\rho} \Rightarrow \varepsilon_{top} = \frac{y_{top}}{\rho} \Rightarrow \rho = \frac{y_{top}}{\varepsilon_{top}} = \frac{50\,mm}{.0008}$$

$$\rho = 6.25\times10^4\,mm = \boxed{62.5\,m = \rho} \quad \boxed{\kappa = \frac{1}{\rho} = 1.6\times10^{-2}\,m^{-1}}$$



since $L = 1.2$ m

$L = \rho \sin\theta \Rightarrow \theta = \sin^{-1}\frac{L}{\rho}$

$\theta = 1.1^\circ$

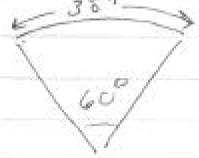
$\delta = \rho - \rho\cos\theta = 11.5$ mm

$$\boxed{\delta = 11.5\,mm}$$

2 (5.5-3) M_0  M_0

ruler has a thickness of 0.1" (rectangular cross-section)

length $L = 30''$ and when bent is forms an arc of $\alpha = 60°$



$\rho\alpha = 30''$ with α in radians

$60°\left[\frac{\pi}{180}\right] = \frac{\pi}{3} \Rightarrow \rho = \frac{30}{\pi/3} = 28.65''$

$\sigma = E\varepsilon = \frac{Ey}{\rho}$ (tensile on top)

$\sigma_{max} = \frac{(29 \times 10^6)(0.1/2)}{28.65}$

$\boxed{\sigma_{max} = 50{,}600 \text{ psi}}$

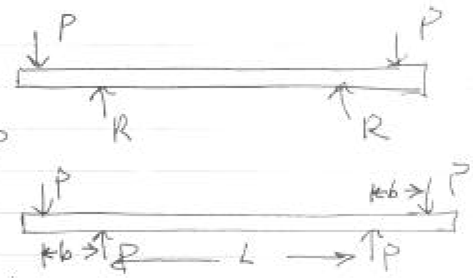
neutral axis is in middle of cross-section

so max strain is where $y = t/2 = \frac{0.1}{2}$

If angle increases (it is more bent), then ρ decreases

$\Rightarrow \boxed{\sigma_{max} \text{ increases}}$

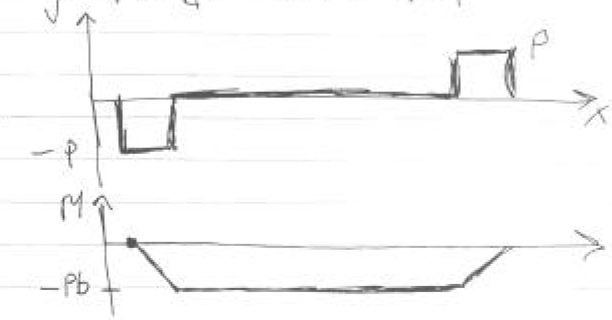
3. (5.5-6)



$\Sigma F_y = 0 \Rightarrow R = P$

Maximum bending moment is
$Pb = (46{,}500)(0.2)$
$= 9300$ N-m

actually it is negative
9300 N



Cross-section is circular with diameter d = 80 mm

$\sigma = \frac{-My}{I}$ $\qquad I = \frac{\pi}{4} r^4$ for circular crossection

$I = \frac{\pi}{4}(.04)^4 = 2 \times 10^{-6}\ m^4$

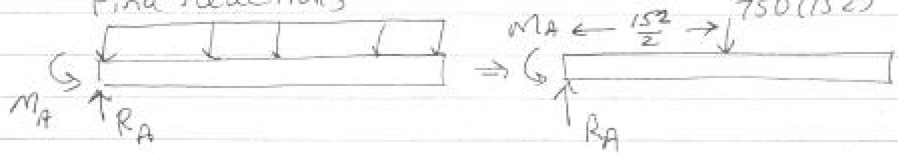
max stress is at top where $y = r = 0.04$ m

$\sigma_{max} = \frac{(9300)(.04)}{2 \times 10^{-6}} = \boxed{185 \text{ mPa} = \sigma_{max}}$

4 (5.5-9)

bending moment is maximum at the support

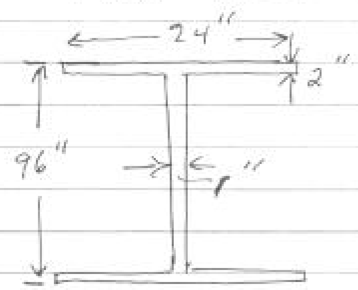
Find reactions



$\Sigma F_y = R_A - 750(152) = 0 \Rightarrow R_A = 1.14 \times 10^5$ lb

$\Sigma m|_{A_z} = M_A - 750(152)\left(\frac{152}{2}\right) = 8.66 \times 10^6$ lb-ft

$M_{max} = 8.66 \times 10^6$ lb-ft $= 1.04 \times 10^8$ lb-in.



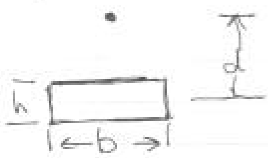
By symmetry (top and bottom), can see that the centroid is in the center.

Need to find $I = \int y^2 dA$ for cross-section.

For a rectangular cross-section

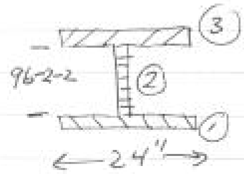
I about the center of the rectangular is $\frac{1}{12} bh^3$

To find I for this rectangle about a point d from the center we use the parallel axis theorem



$$I = \frac{1}{12} b h^3 + (Area) d^2$$
$$= \frac{1}{12} b h^3 + bhd^2$$

Break I beam into 3 parts (each rectangular)



we want I for each part taken about the center of the whole area (centroid) since centroid of area 2 coincides with centroid of whole area, we just use $I_{2} = \frac{1}{12} b h^3 = \frac{1}{12}(1)(92)^3 = 6.49 \times 10^4 \; in^4$

For areas 1 and 3 we use parallel axis theorem in both cases, $b = 24''$, $h = 2''$, $d = 1 + 46 = 47''$
(d is distance from center of 1 to center of whole figure)

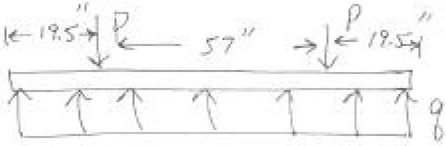
$$I_{1} = I_{3} = \frac{1}{12}(24)(2)^3 + (24)(2)(47)^2 = 1.06 \times 10^5 \; in^4$$
$$I = I_{1} + I_{2} + I_{3} = 2.77 \times 10^5 \; in^4$$

max value of y is from center to top or bottom

$y_{max} = 48''$

$$\sigma_{max} = \frac{M_{max}\, y_{max}}{I} = \frac{(1.04 \times 10^8)(48)}{2.77 \times 10^5} = \boxed{18{,}000 \text{ psi} = \sigma_{max}}$$

5. (5.5-11)



$P = 36{,}000$

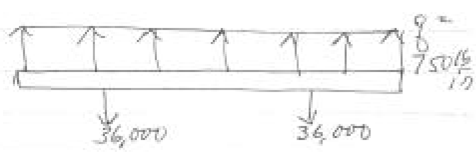
First find the distributed force q which maintains equil.

$\Sigma F_y = -36{,}000 - 36{,}000 + q(19.5 + 19.5 + 57) = 0$

$q = 750$ lb/in

Find V & M diagrams

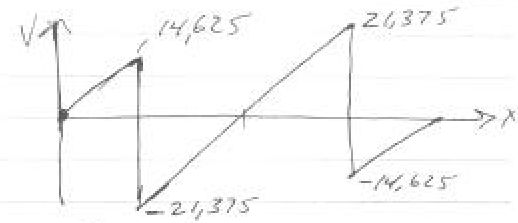
(redraw q on top to make more familar, but doesn't matter)



$(750)(19.5) = 14625$ lb

$14625 - 36000 = -21{,}375$ lb

$-21{,}375 + 750(57) = 21{,}375$



$M(0) = 0$

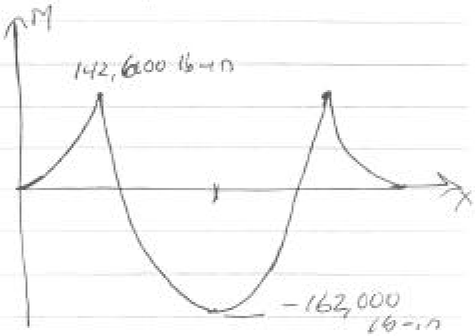
$M(19.5) = \frac{1}{2}(14{,}625)(19.5) =$

$= 142{,}600$ lb-in

$V = 0$ at center (48" from end)

$M(48) = 142{,}600 - \frac{1}{2}(21{,}375)(\frac{57}{2})$

$= -162{,}000$ lb-in



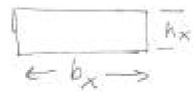
max stress on top or bottom same (since symmetric)

use $|M_{max}| = 162{,}000$ lb-in

$$\sigma_{max} = \frac{162{,}000 \; h/2}{\frac{1}{12} b h^3} = \frac{6(162{,}000)}{(12)(10)^2} = \boxed{810 \text{ psi} = \sigma_{max}}$$

Extra problem (5.7-9)

Beam has a varying cross-section.
where b_x and h_x vary with x.



But Bending moment also varies with x.
Want same maximum stress along whole beam.

Find $M(x)$

$$\sum M\Big|_{cut_2} = qx\left(\frac{x}{2}\right) + M = 0 \Rightarrow M = -qx^2/2$$

At any cross-section x, the stress is $\frac{-My}{I} = \sigma$

At top, $y = \frac{h_x}{2}$, $I = \frac{1}{12} b_x h_x^3$

$$\sigma = \frac{\frac{qx^2}{2} h_x/2}{\frac{1}{12} b_x h_x^3} = \text{constant}$$

we are given that $b_x = b_B \frac{x}{L}$

$$\frac{3qx^2 h_x}{b_B \frac{x}{L} h_x^3} = \text{const} \quad \text{or} \quad \frac{x^2}{x\, h_x^2} = \text{const}$$

$$\text{or} \quad h_x^2 \propto x \quad \text{or} \quad h_x \propto \sqrt{x}$$

so if h_B is value at $x = L$, then

$$\boxed{h_x = h_B \sqrt{\frac{x}{L}}}$$