Solutions to Set #11, 24-261, Fall 2001

1. (3.7-9)

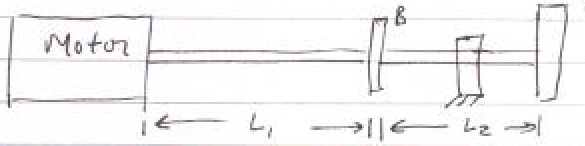
Motor delivers 275 hp at 1000 rpm

Find torque delivered by motor

Power $= 275\ hp = 275(550)\ \frac{ft\text{-}lb}{s} = 151 \times 10^3\ \frac{ft\text{-}lb}{s}$

$1000\ rpm = 1000\ \frac{2\pi\ rad}{rot}\ \frac{1\ min}{60\ s} = 104.7\ rad/s$

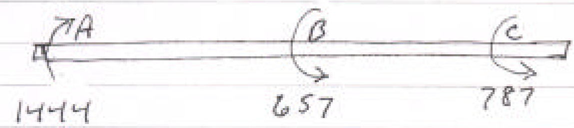
Since $P = T\omega \Rightarrow T = \frac{P}{\omega} = \frac{151 \times 10^3}{104.7} = 1444\ ft\text{-}lb$



since whole shaft has $\omega = 104.7\ rad/s$,
if gears B & C deliver 125 and 150 hp, respectively
They apply torques of $T_B = \frac{(125)(550)}{104.7} = 657\ ft\text{-}lb$

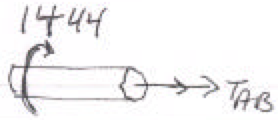
and $T_C = \frac{(150)(550)}{104.7} = 787\ ft\text{-}lb$

Look at shaft



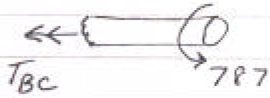
shaft is running at constant speed; it is
in equilibrium (torques balance)

In AB



$T_{AB} = 1444$

In BC



$T_{BC} = 787$

Max shear stress is in AB (T is max there and cross-section is uniform)

$$\tau_{max} = \frac{Tr}{I_p} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3}$$

$$\frac{2T}{\pi r^3} < 7500 \Rightarrow r > \left[\frac{2T}{\pi(7500)}\right]^{1/3} = \left[\frac{2(1444)(12)}{\pi(7500)}\right]^{1/3}$$

$$r > 1.137''$$

Total angle of twist = $\Delta\varphi_{AB} + \Delta\varphi_{BC} < 1.5^\circ$

$$\frac{T_{AB}L_1}{GI_p} + \frac{T_{BC}L_2}{GI_p} < 1.5^\circ \Rightarrow (1444)(6)(12)^2 + (787)(4)(12)^2 < 1.5\left(\frac{\pi}{180}\right)(11.5\times10^6)\frac{\pi}{2}r^4$$

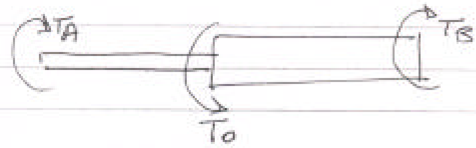
$$\Rightarrow r > 1.377''$$

Both conditions must be satisfied

r must be greater than maximum

$$\Rightarrow \boxed{d > 2.75''}$$

2. (3.8-5)



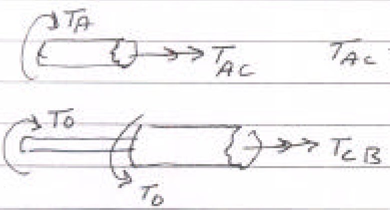
$\Sigma M|_x = T_0 - T_A - T_B = 0 \Rightarrow T_A + T_B = T_0$ static. indet.

Know that ends A & B are fixed $\Rightarrow \varphi_A = \varphi_B = 0$

twist of ~~AB~~ $= \varphi_C - \varphi_A \Rightarrow \Delta\varphi_{AC} = \varphi_C \quad (\varphi_A = 0)$

twist of $CB = \varphi_B - \varphi_C \Rightarrow \Delta\varphi_{CB} = -\varphi_C \quad (\varphi_B = 0)$

So twists are related $\Delta\varphi_{AC} = -\Delta\varphi_{CB}$



$T_{AC} = T_A$

$T_{CB} = T_A - T_0$

Apply equil. to find internal torques

~~Apply torque-twist relation~~

$$\Delta\varphi_{AC} = \frac{T_{AC} L_{AC}}{G I_{P_{AC}}} \qquad \Delta\varphi_{CB} = \frac{T_{CB} L_{CB}}{G I_{P_{CB}}}$$

$$I_{P_{AC}} = \frac{\pi}{2}(0.4)^4 \qquad I_{P_{CB}} = \frac{\pi}{2}(0.8)^4$$

$$\Delta\varphi_{AC} = -\varphi_{CB} \Rightarrow \frac{T_A(8)}{(0.4)^4} = -\frac{(T_A - T_0)(20)}{(0.8)^4}$$

$$T_A(8)(2)^4 = (T_0 - T_A)(20) \Rightarrow T_A = \frac{20\,T_0}{20 + 8(16)}$$

or $T_A = 0.135\, T_0$

$$T_A + T_B = T_0 \Rightarrow T_B = 0.865\, T_0$$

Consider max shear stress in each

$$(\tau_{max})_{AC} = \frac{|T_{AC}|\, r_{AC}}{\frac{\pi}{2}(r_{AC})^4} \Rightarrow \frac{2(0.135)T_0}{\pi(0.4)^3} = 1.343\, T_0$$

$$(\tau_{max})_{CB} = \frac{|T_{CB}|\, r_{CB}}{\frac{\pi}{2}(r_{CB})^4} = \frac{2(0.865)T_0}{\pi(0.8)^3} = 1.076\, T_0$$

so max shear is in AC

$$1.343\, T_0 = 8000\ \text{psi} \Rightarrow \boxed{T_0 = 5960\ \text{lb-in}}$$

$$\Rightarrow T_B = 0.865\, T_0 = 5150\ \text{lb-in}$$

In CB the stress is governed by $|T_{CB}|$

$$\tau = \frac{T\rho}{I_p} = \frac{(5150)(0.5)}{\frac{\pi}{2}(0.8)^4} = 4000\ \text{psi}$$

$\boxed{\tau = 4000\ \text{psi}}$ at a radial position of 0.5" from center-line in the segment CB

$$(\varphi_B - \varphi_D) = \frac{TL}{GI_p} = \frac{(-5150)(10)}{(11\times10^6)\frac{\pi}{2}(0.8)^4} = -7.28\times10^{-3}$$

since $\varphi_B = 0$ $\boxed{\varphi_D = 7.28\times10^{-3}\ \text{rad} = 0.417^\circ}$

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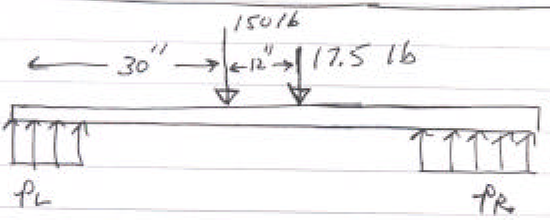
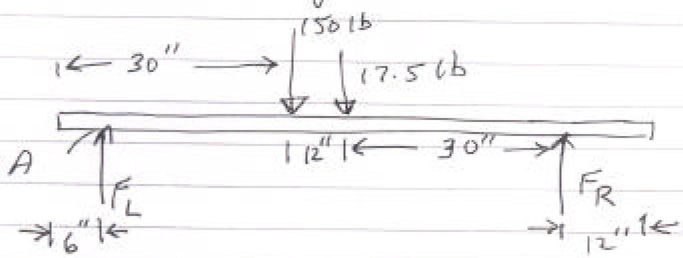
Weight of board = $\left(40 \frac{lb}{ft^3}\right)\left(\frac{1 ft}{12 in}\right)^3 (84)(0.75)(12)$

$W = 17.5 \text{ lb}$



17.5 acts at midpoint of beam (since we only want net force + moment to be right)

Pressures are uniform ⇒ replace by pressure times area acting in middle. since we are only interested in net force & moment



$\Sigma M|_{A_z} = (-150)(24) - (17.5)(36) + F_R(66) = 0$

$F_R = 64.1 \text{ lb}$

$\Sigma F_y = F_L + F_R - 150 - 17.5 = 0 \Rightarrow F_L = 103.4 \text{ lb}$

$\boxed{p_L = \frac{103.4}{(12)(12)} = 0.718 \text{ psi}}$ $\boxed{p_R = \frac{64.1}{(12)(24)} = 0.223 \text{ psi}}$

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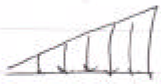


$q(x) = 50\frac{x}{9}$

Force acting on dx is $q(x)dx = 50\frac{x}{9}dx$

(i) Net force $= \int_0^9 50\frac{x}{9}\,dx = 50\frac{x^2}{18}\Big|_0^9$

$= 225\ N$

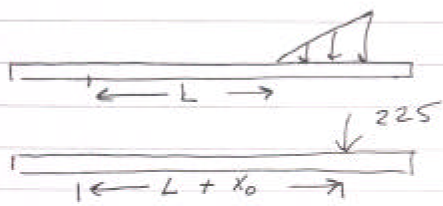
(ii) Net moment is found from adding contribution of each dx to M/O_z

$$\underbrace{(L+x)}_{\text{moment arm}}\underbrace{q(x)\,dx}_{\text{force}}$$

Net moment $= \int_0^9 [L+x]\,50\frac{x}{9}\,dx$

$= 50L\frac{x^2}{18}\Big|_0^9 + \frac{50x^3}{27}\Big|_0^9 = 225L + 1350\ N$

(iii)



$(L+x_0)(225) = 225L + 225x_0 = 225L + 1350$

$\Rightarrow x_0 = 6$ cm

so force is to be applied at $x = 6$ cm.

This $\frac{2}{3}$ of way from beginning of distribution to the end.

$$F_t = 100\cos 30^\circ - F_A - F_B$$
$$= 100\cos 30^\circ - \mu_K (N_A + N_B)$$
$$= 100\cos 30^\circ - \mu_K (10)\left[17 - 4\mu_K\right]$$

(a) $\mu_K = 0 \Rightarrow F_t = 100\cos 30^\circ = 86.6$ lb

(b) $\mu_K = 0.1 \Rightarrow F_t = 70$ lb

if sleeve is thicker, then the forces N_A & N_B would be smaller. Hence F_t would be closer to $100\cos 30^\circ$

if sleeve is thinner, N_A & N_B would be larger. F_t would even less compared to $100\cos 30^\circ$