Solutions to Problem Set #10, 24-261, Fall 2001

1 (3.2-3)

Let bar be length L

Let twisting be described by

no rotation at left, rotation of φ at right

Derived in class $\gamma = \frac{\rho\varphi}{L}$

at $\rho = r_1$ inner surface, $\rho = r_2$ outer surface

(a) max strain at outer: $\gamma_{max} = 350 \times 10^{-6} = \frac{r_2 \varphi}{L}$

$\Rightarrow \frac{\varphi}{L} = \frac{350 \times 10^{-6}}{r_2}$

At inner radius, $\gamma_1 = \frac{r_1 \varphi}{L} = 350 \times 10^{-6} \frac{r_1}{r_2}$

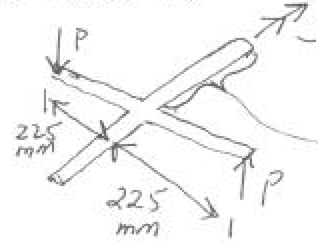
$\frac{r_1}{r_2} = 0.5 \Rightarrow \boxed{\gamma_1 = \frac{1}{2}(350 \times 10^{-6}) = 175 \times 10^{-6}}$

(b) $\frac{\varphi}{L} = 0.2\left(\frac{deg}{ft}\right)\left(\frac{\pi}{180}\frac{rad}{deg}\right)\left(\frac{1 ft}{12 in}\right) = 2.91 \times 10^{-4} \frac{rad}{in}$

$\gamma_{max} = 350 \times 10^{-6} = \frac{r_2 \varphi}{L} \Rightarrow r_2 = \frac{350 \times 10^{-6}}{\varphi / L}$

$\boxed{r_2 = 1.20''}$

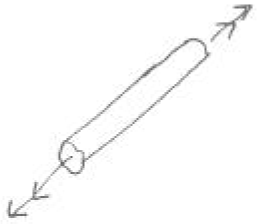
2 (3.3-4)



moment of lug nut on wrench

$P = 100\ N$

This part of the wrench is twisting



$$T = P(450\ mm) = 45\ N\text{-}m$$

Solid cross-section

diameter is $d = 12\ mm$

shear modulus $G = 78\ GPa$

max shear stress is $\tau_{max} = \frac{Tr}{I_p}$

$$I_p = \frac{\pi}{2} r^4 \Rightarrow \tau_{max} = \frac{Tr}{\frac{\pi}{2} r^4} = \frac{2T}{\pi r^3}$$

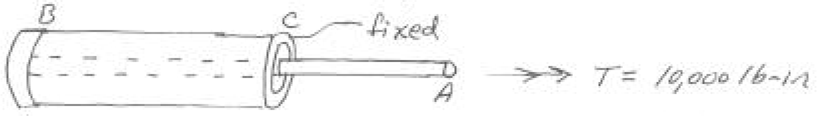
$$\tau_{max} = \frac{2(45)}{\pi(.006)^3} = 133\ MPa$$

$$\Delta\varphi = \frac{TL}{G I_p} = \frac{(45)(.225)}{(78\times10^{9})\frac{\pi}{2}(.006)^4} = 6.376\times10^{-2}\ rad$$

$$\boxed{\Delta\varphi = 6.376\times10^{-2}\left(\frac{180}{\pi}\right) = 3.65^{\circ}}$$

twist or relative rotation of wrench

3 (3.4-3) Both outer tube + inner rod twist.



End plate keeps tube + rod rotating at same angle at B.

tube is fixed against rotation at end C

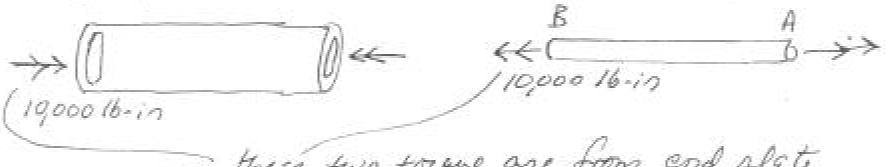
Bar AB is 40" long, $d_1 = 1.60''$,

Tube BC is 20" long, $\underbrace{d_2 = 2.35''}_{\text{Inner diam}}$, $\underbrace{d_3 = 2.75''}_{\text{outer diameter}}$

Both materials have $G = 3.9 \times 10^6$ psi

$\varphi_C = 0$, φ_B same for AB & BC, find φ_A

By equilibrium both experience same torque



these two torque are from end plate must be equal + opposite

Consider tube first since $\varphi_c = 0$

In this tube, $T = -10,000$ lb-in

(since it is

)

$$\underset{\substack{\text{right}\\ \text{end}}}{\varphi_C} - \underset{\substack{\text{left}\\ \text{end}}}{\varphi_B} = \frac{TL}{GI_p} = \frac{-10,000(20)}{(3.9\times10^6)\frac{\pi}{2}\left[\left(\frac{2.75}{2}\right)^4-\left(\frac{2.35}{2}\right)^4\right]}$$

$$= -1.957\times10^{-2} \text{ rad}$$

since $\varphi_c = 0 \Rightarrow \varphi_B = +1.957\times10^{-2}$ rad

Solid shaft Now $T = 10,000$ lb-in

$$\varphi_A - \varphi_B = \frac{TL}{GI_p} = \frac{10,000(40)}{(3.9\times10^6)\frac{\pi}{2}\left(\frac{1.6}{2}\right)^4} = 1.594\times10^{-1} \text{ rad}$$

$$\varphi_A = \varphi_B + 1.594\times10^{-1} = 1.790 \,.\, \times10^{-1} \text{ rad}$$

$$\boxed{\varphi_A = 1.790\times10^{-1}\left(\frac{180}{\pi}\right) = 10.3^0}$$

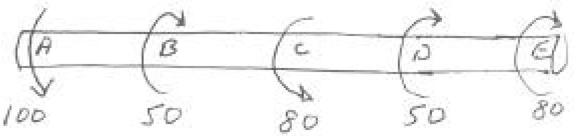
In bar $\tau = \frac{Tr}{I_p} = \frac{Tr}{\frac{\pi}{2}r^4} = \frac{2T}{\pi r^3} = \frac{2(10,000)}{\pi(0.8)^3}$

$$\boxed{\tau_{bar} = 12,430 \text{ psi}}$$

In tube $\tau = \frac{T\ r_3}{\frac{\pi}{2}(r_3^4 - r_2^4)}$

$$\tau_{tube} = \frac{10,000\left(\frac{2.75}{2}\right)}{\frac{\pi}{2}\left[\left(\frac{2.75}{2}\right)^4-\left(\frac{2.35}{2}\right)^4\right.} = \boxed{5,250 \text{ psi} = \tau_{tube}}$$

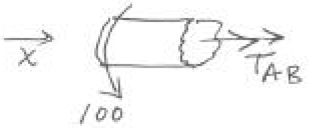
4 (3.4-4)



All torques in N-m

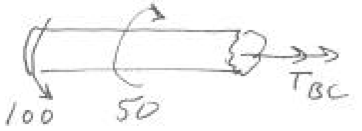
Find internal torques. Draw FBD from left end to a cross-section of interest

cross-section between A & B



$\Sigma M|_x = 100 + T_{AB} = 0 \Rightarrow T_{AB} = -100$ N-m

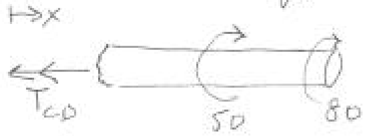
cross-section between B & C



$\Sigma M|_x = 100 - 50 + T_{BC} = 0$

$T_{BC} = -50$ N-m

cross-section between C & D
(For variety, will isolate from section to right end)

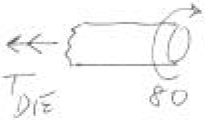


$\Sigma M|_x = -T_{CD} - 50 - 80 = 0$
(consistently summing moments about +x-axis)

$\Rightarrow T_{CD} = -130$ N-m

Notice how T_{CD} is drawn in direction which is positive by convention

Cross-section between D & E



$$\Sigma M|_x = -T_{DE} - 80 = 0$$

$$T_{DE} = -80 \text{ N-m}$$

Conditions on max shear stress and twist per unit length depend on $|T|$. Max value of $|T|$ along shaft is between C & D $|T|_{max} = 130$ N-m

$$\tau_{max} = \frac{T r_2}{I_p} = \frac{T r_2}{\frac{\pi}{2}(r_2^4 - r_1^4)}$$

[Rod has known outer radius r_2, we want inner radius r_1]

$\tau_{max} < 80$ mPa $\Rightarrow$

$$\frac{T r_2}{\frac{\pi}{2}(r_2^4 - r_1^4)} < 80 \text{ mPa} \Rightarrow r_1 < \left[r_2^4 - \frac{2 T r_2}{\pi (80 \times 10^6)} \right]^{1/4}$$

$$r_2 = \frac{25}{2} \text{mm} = 12.5 \times 10^{-3} \text{m} \; ; \; T = 130 \text{ N-m}$$

$$\Rightarrow \quad r_1 < 10.35 \text{ mm} \Rightarrow d_1 < 20.7 \text{ mm}$$

$$\text{Since } \Delta\varphi = \frac{TL}{GI_p} \Rightarrow \frac{T}{GI_p} = \frac{\Delta\varphi}{L} < 6 \frac{\text{deg}}{\text{m}} = 0.1047 \frac{\text{rad}}{\text{m}}$$

$$\frac{T}{G\frac{\pi}{2}[r_2^4 - r_1^4]} < 0.1047 \Rightarrow r_1 < \left[r_2^4 - \frac{2T}{\pi G (0.1047)} \right]^{1/4}$$

For monel, $G = 66$ GPa, $r_2 = 12.5 \times 10^{-3}$ m, $T = 130$ N-m

$$r_1 < 10.56 \text{ mm} \Rightarrow d_1 < 21.1 \text{ mm}$$

Both conditions must hold $\Rightarrow d_1$ is governed by lesser of the two $\boxed{(d_1)_{min} = 20.7 \text{ mm}}$