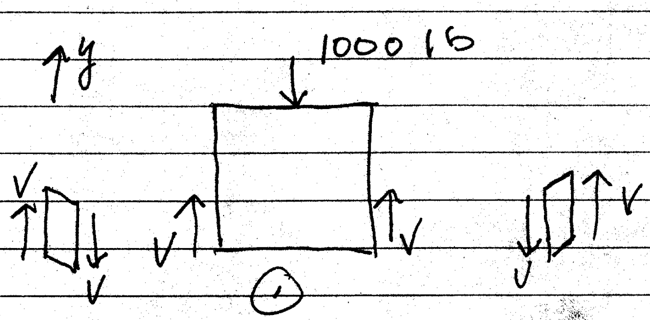
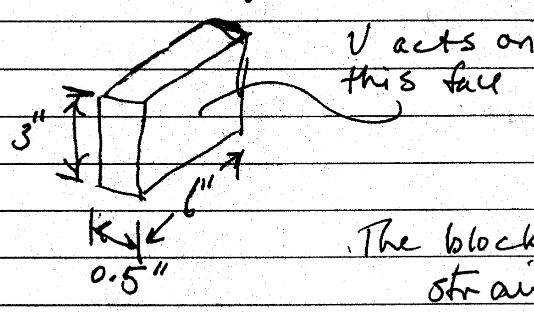
solutions for practice problems

1.



(1) $\Sigma F_y = -1000 + 2V = 0 \Rightarrow V = 500$ lb



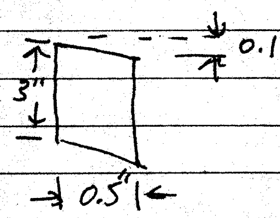
$\tau = \frac{V}{A} = \frac{500}{(3)(6)}$

$=$

The block shears with a strain

$\gamma = \frac{0.1}{0.5}$

$= 0.2$



The shear modulus G relates τ & γ

$\tau = G\gamma \Rightarrow G = \frac{\tau}{\gamma} = \frac{500/18}{0.2}$

$\boxed{G = 139 \text{ psi}}$

2.

Treat the strut is effectively rigid relative to cable (Area is 200 times that of cable)

Cable + strut are two-force members

$\Sigma F_y = -500 + P_{BC} \sin 30^\circ = 0$

$P_{BC} = 1000\ N$

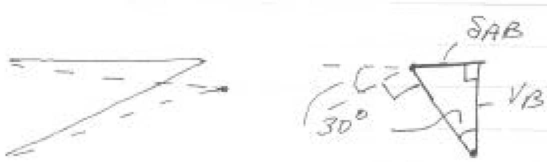
$\Sigma F_x = -T_{AB} + P_{BC} \cos 30^\circ$

$\Rightarrow T_{AB} = 1000 \cos 30^\circ = 866\ N$

Find stretch of AB $\Rightarrow$ " $\delta = \frac{PL}{EA}$ "

$$\delta_{AB} = \frac{(866)(0.8)}{(200\times10^{9})(5\ \times10^{-6})} = 0.693\ mm$$

BC is rigid $\Rightarrow$ it pivots about C, point B moves $\perp$ to CB since motion is small



$V_B = \frac{\delta_{AB}}{\tan 30^\circ} = 1.2\ mm$

B moves 0.693 mm to right
B " 1.2 mm down

The force carried by pin at C
is 1000 N

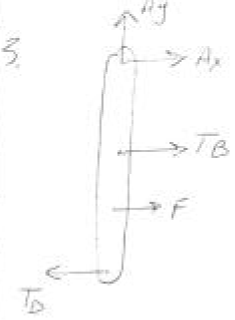
from strut BC

$V = 1000 \text{ N}$

$$\tau = \frac{V}{A} = \frac{1000}{\frac{\pi}{4}(.004)^2} = 79.6 \text{ MPa}$$

$$\boxed{\tau = 79.6 \text{ MPa}}$$

3.



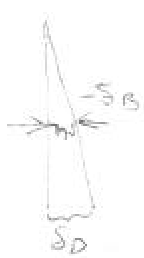
$\Sigma M_A = -T_D(400) + T_B(200) + F(300) = 0$

$4T_D - 2T_B = 3F$

statically indeter. ($\Sigma F = 0$ would give two more eqns for A_x, A_y)

Relate T_B, T_D to stretches of those bars. Then relate displacements at B + D (since they are part of a rigid bar)

$\delta_B = \dfrac{T_B(.2)}{E(600\times10^{-6})}$ $\qquad \delta_D = \dfrac{T_D(.15)}{E(400\times10^{-6})}$



$\dfrac{-\delta_B}{200} = \dfrac{\delta_D}{400} \Rightarrow \delta_D = -2\delta_B$

$\Rightarrow \dfrac{T_D(.15)}{E(400\times10^{-6})} = \dfrac{-2T_B(.2)}{E(600\times10^{-6})}$

$\Rightarrow T_D(6)(.15) = -2T_B(4)(.2)$

$T_D = -1.778\,T_B$

$-4(1.778)T_B - 2T_B = 3F$

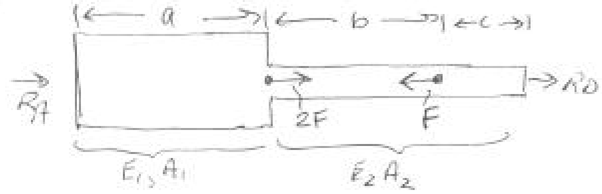
$\Rightarrow T_B = -.329\,F \qquad T_D = 0.585\,F$

$\sigma_B = \dfrac{T_B}{A} = \dfrac{-.329F}{600\times10^{-6}} = -50\times10^{6} \Rightarrow F = 91{,}200\ N$

$\sigma_D = \dfrac{T_D}{A} = \dfrac{.585F}{400\times10^{-6}} \Rightarrow F = 34{,}200\ N$

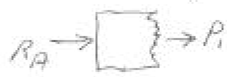
Keep both links below 50 MPa $\Rightarrow$ **$F_{allow} = 34{,}200\ N$**

4.



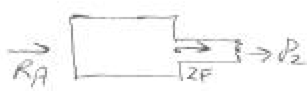
$$\Sigma F_x = R_A + 2F - F + R_D = 0 \Rightarrow R_A + R_D = -F$$

$0 < x < a$ (segment 1)



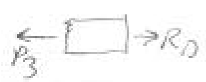
$$\Sigma F_x = R_A + P_1 = 0 \Rightarrow P_1 = -R_A \; ; \; \delta_1 = \frac{P_1 a}{E_1 A_1} = \frac{-R_A a}{E_1 A_1}$$

$a < x < a+b$ (segment 2)



$$\Sigma F_x = R_A + 2F + P_2 = 0 \Rightarrow P_2 = -R_A - 2F \; ; \; \delta_2 = \frac{P_2 b}{E_2 A_2} = \frac{-(R_A + 2F)(b)}{E_2 A_2}$$

$a+b < x < a+b+c$ (segment 3)



$$\Sigma F_x = -P_3 + R_D = 0 \Rightarrow P_3 = R_D = -R_A - F \qquad \delta_3 = \frac{P_3 c}{E_2 A_2} = \frac{-(R_A + F)(c)}{E_2 A_2}$$

Total change in length is 0 $\rightarrow \delta_1 + \delta_2 + \delta_3 = 0$

$$\Rightarrow R_A \left[\frac{a}{E_1 A_1} + \frac{b}{E_2 A_2} + \frac{c}{E_2 A_2} \right] = \frac{-2Fb}{E_2 A_2} - \frac{Fc}{E_2 A_2}$$

$$R_A = -F \left[\frac{2b + c}{\frac{E_2 A_2}{E_1 A_1} a + b + c} \right] ; \; R_D = -R_A - F$$

$$R_D = F \left[\frac{2b + c}{\frac{E_2 A_2}{E_1 A_1} a + b + c} - 1 \right] = F \left[\frac{b - \frac{E_2 A_2}{E_1 A_1} a}{\frac{E_2 A_2}{E_1 A_2} a + b + c} \right]$$

Midway along segment of length b internal force is P_2

$$\sigma = \frac{P_2}{A_2} = \frac{1}{A_2}\left\{-R_A - 2F\right\} = \frac{1}{A_2}\left\{F\left[\frac{2b+c}{\frac{E_2 A_2}{E_1 A_1}a+b+c} - 2\right]\right\}$$

$$\sigma = \frac{F}{A_2}\left\{\frac{-c - 2\frac{E_2 A_2}{E_1 A_1}a}{\frac{E_2 A_2}{E_1 A_1}a + b + c}\right\}$$

For segment $C \to D$ $\qquad \delta_3 = u_D - u_C \qquad$ but $u_D = 0$

$$u_C = -\delta_3 = \frac{(R_A + F)c}{E_2 A_2} = \frac{-R_D c}{E_2 A_2}$$

$$u_C = \frac{-Fc}{E_2 A_2}\left\{\frac{b - \frac{E_2 A_2}{E_1 A_1}a}{\frac{E_2 A_2}{E_1 A_1}a + b + c}\right\}$$