

## Topics Covered in Lecture

These notes are intended to serve as a reminder of the key points covered in lecture. They are a supplement to lecture and do not replace it.

### Lecture 1 (August 28):

- ✍ General introduction to course
- ✍ Definitions of directions of translation and rotation in three dimensions. Rotations are defined using the right hand rule, with the thumb pointing along the axis and the fingers curling in the direction of rotation.
- ✍ Definition of Forces and Moments (that is, directly applied moments) in three dimensions. Think of forces and moments as resisting motion. Use of symbols  $F_x$ ,  $F_y$ ,  $F_z$ ,  $M_x$ ,  $M_y$ , and  $M_z$ . A directly applied moment is one body grabbing another and tending to rotate it. This is distinct from the tendency of a force to cause a body to pivot about some axis (taken up in Lecture 3).

### Lecture 2 (August 30):

- ✍ Consider a force due, for example, to a cable. The force acts along the cable. The direction is known in terms of the end points of the cable. You want the x-y-z components of the force. Find the vector running from one end of the cable to another (units of length). Divide vector by its magnitude – this produces the unit vector in the direction of the cable force. Multiply by magnitude of the cable force if known, or by a variable which represents the magnitude of the cable force which is to be determined.
- ✍ Use various methods of trigonometry to determine distances and angles. Methods include law of cosines, law of sines, and x-y components

### Lecture 3 (September 4):

- ✍ Definitions of distributed forces: force per unit volume, force per unit area, and force per unit length.
- ✍ The rotational tendency of a force. Besides directly applied moments (Lecture 1), forces can also cause a body to rotate. Consider a given force applied at one point and consider an x-, y- or z-axis through some other point. Does the force tend to cause rotation about the axis? The sign of the moment is defined by the right hand rule. The magnitude of the moment depends on the magnitude of the force and on the perpendicular distance.
- ✍ A force whose line of action runs through the axis produces no moment. A force that acts parallel to the axis produces no moment. A force may be slid along its line of action with no change in the moment it produces about any axis.
- ✍ One can break a force into components, find the moment due to each component individually, and then sum them. To think about the perpendicular distance, the

force component in question must have a line of action perpendicular to the axis about which rotation is considered. For two such perpendicular lines, one can find the unique segment which is perpendicular to both with end points on those lines. The length of this segment is the “perpendicular distance”.

- ✍ For forces lying in a plane (say x-y), moments about the perpendicular axis (z-axis) can be found by a variety of methods. They are: finding the perpendicular force component, finding the perpendicular distance, evaluating  $rF\sin\theta$ , and finding moments due to x-y components and adding them up. In any event, one must determine the sign of the moment by thinking about the direction of rotation and using the right hand rule.

#### Lecture 4 (September 6):

- ✍ Equations of equilibrium. Once a body has been chosen for study (it could be a distinct physical part, a collection of parts, or subset of a part), all the forces and directly applied moments acting on it must be identified. The free body diagram is a diagram which displays the body and all forces and directly applied moments acting on the body. Some of the forces or directly applied moments may be unknown; they are represented by symbols rather than by actual magnitudes. The moments due to forces are not included in the free body diagram.
- ✍ The equations of equilibrium state that the net tendency to accelerate is zero (the body is stationary). When the total force is zero in each direction, there is no tendency to translate. When the total moment is zero about each axis, there is no tendency to rotate. In computing the total moment, one must include both directly applied moments and moments due to forces. Besides situations in which the body is stationary, one can also apply the equations of equilibrium when the body is translating or rotating at constant speed.
- ✍ Only after a body is clearly chosen and its free body diagram is completed should one consider the equations of equilibrium. The free body diagram is a way of being clear that all the external influences are included. For every force and directly applied moment that appears in the free body diagram, one must be able to identify its source. That is, some body external to the body of the free body diagram is exerting that force or directly applied moment. (The body of the free body diagram exerts an equal and opposite force or moment back on the source. If the source is also studied, then that reaction must be included in its free body diagram.) For most mechanical engineering purposes, one body applies a force to another by contacting it. The only exception is the earth which exerts a gravitational influence on all bodies.
- ✍ The most general way in which two bodies can interact is through a force in each direction and through a moment in each direction. By most general, we mean this is the largest number of unknown force and moment components which one needs to consider when separating two interacting bodies.

#### Lecture 5 (September 13):

- ✍ Connections. Two bodies are often connected together mechanically in complex ways. These connections are also sometimes called joints. Joints permit the bodies to move with respect to one another in some directions and not in others. For a given connection, one should consider relative translations of the two bodies and relative rotations of the two bodies. By breaking into x-y-z components, one can usually state unambiguously whether translations and rotations along each of the coordinate directions can occur or not. (There are some odd exceptions such as a screw joint, in which the rotation and translation are coupled. Such exceptions are not of concern in this class.)
- ✍ When the two bodies cannot be translated along a given axis relative to one another, then each can exert a force on each other along that axis. When the two bodies cannot be rotated about a given axis relative to one another, then each can exert a direct moment on each other about that axis.
- ✍ When there is light frictional resistance to a given motion the respective force or moments is often neglected.
- ✍ When no relative motion is possible between two bodies (they are rigidly connected), then can exert upon one another a force in each direction and a moment in each direction.
- ✍ When one considers the detailed way in which two distinct bodies interact, one finds that at each point of contact there can be a force (not a directly applied moment). When the force acts normal to the plane of contact, then the force is termed a normal force.
- ✍ When we say that two bodies apply a moment directly upon one another, this is a useful short hand for a collection of two or more contact forces, which together result in a moment of each body on the other.
- ✍ For example, two equal and opposite forces a distance apart produce a net moment equal to the force magnitude times the separation distance between the forces. The moment is about the axis which is mutually perpendicular to both the direction of the forces and the perpendicular between them. The moment is the same about all points.
- ✍ Sometimes a directly applied moment is, in fact, produced by two such equal and opposite forces. Sometimes a directly applied moment is a consequence of a more complex combination of forces.

#### Lecture 6 (September 20):

- ✍ Statically equivalent loads. A combination of forces can sometimes be reduced to a single force which produces the same net force and net moment about any point. The original loads and the load that produces it are termed statically equivalent loads.
- ✍ The force of gravity acts on every piece of a body. Those forces produce a net force and a net moment about any chosen point. One can produce the same net force and moment by having a single force equal to the total weight applied at a point called the center of gravity. This in fact is the definition of the center of gravity.

- ~~✍~~ Example of a statics problem involving several connected members. We consider the way in which two bodies can move relative to one another to deduce what forces and direct moments each can exert on the other. When we separate the bodies, we draw these forces and moments as unknowns, equal and opposite on the two bodies.
- ~~✍~~ When a body has a force in any direction applied to one point and a force in any direction applied at a second point and no other loads, this is termed a two-force body or two-force member. The two forces must be equal and opposite and they must act along the line connecting the two points. The magnitude and sense of the two forces cannot be determined from that member alone; other bodies must be considered.
- ~~✍~~ This problem involved several bodies and hence several free body diagrams. One should normally only apply the equations of equilibrium to a body when the free body diagram is complete.