ME 24-221
Thermodynamics I

Solution for the Final Exam
Fall 2000
December 19, 2000

1.

Given: 5kg of water;
State 1: P_1 = 100 kPa, T_1 = 15°C (= 288.15 K)
State 2: T_2 = 60°C (=333.15 K)
Surroundings at 25°C (298.15 K)
500 kJ of work is done on the water, which remains liquid throughout the process

Water is in sub-cooled state in State 1.
Since it remains in liquid state throughout the process 1-2, one can use the value of *C* (the specific heat for water = 4.18 kJ/kgK) to calculate the internal energy changes etc.

From FLT for control mass, ${}_1\Delta U_2 = Q_{1-2} - W_{1-2}$
Where ${}_1\Delta U_2 = mC\Delta T = (5)(4.18)(45) = 940.5$ kJ
$W_{1-2} = -500$ kJ
Hence $Q_{1-2} = {}_1\Delta U_2 + W_{1-2} = 940.5 + (-500) = 440.5$ kJ---(a)

Entropy change in water is given by $\Delta S = mC\ln\left(\frac{T_2}{T_1}\right) = (5)(4.18)(\ln\left(\frac{333.15}{288.15}\right)) = 3.033$ kJ/K ---(b)

Entropy change in the surroundings is given by $\Delta S = \frac{Q_{surr}}{T_{surr}} = \frac{-440.5}{298.15} = -1.4774$ kJ/K ----------------(c)

Entropy change in the universe is $\Delta S_{univ} = \Delta S_{water} + \Delta S_{surr} = 3.033 - 1.4774 = 1.5556$ kJ/K ------(d)

$\Delta S_{univ} > 0$; Thus this process is possible ---(e)

2.

Given: Heavily insulated Piston-Cylinder
3 kg of water
State 1: 100 kPa and *x*=0.8
State 2: 800 kPa
Process 1-2: Irreversible compression with S_{gen}=1.544 kJ/K

FLT for control mass, ${}_1\Delta U_2 = Q_{1-2} - W_{1-2}$; $Q_{1-2} = 0$ (insulated)
Therefore, $W_{1-2} = m(u_1 - u_2)$ --(a)

SLT is ${}_1\Delta S_2 = \int_1^2 \frac{dQ}{T} + S_{gen}$; $Q_{1-2} = 0$

Therefore, $m(s_2 - s_1) = S_{gen}$ --(a)

From Table B.1.1
$s_1 = s_{f1} + x_1 s_{fg1} = 1.3025 + (0.8)(6.0568) = 6.14794$ kJ/kgK

From SLT using Table B.1.1, $s_2 = \frac{S_{gen}}{m} + s_1 = \frac{1.544}{3} + 6.145794 = 6.6627$ kJ/kgK

At P_2 = 800 kPa, s_g = 6.6627 kJ/kg/K;

Since $s_2 = s_{g,800kPa}$, water is at saturated vapor state at state 2.

Hence $T_2 = T_{sat,800kPa} = 170.43^oC = 443.58$ K ---(b)

From FLT, $W_{1-2} = m(u_1 - u_2)$

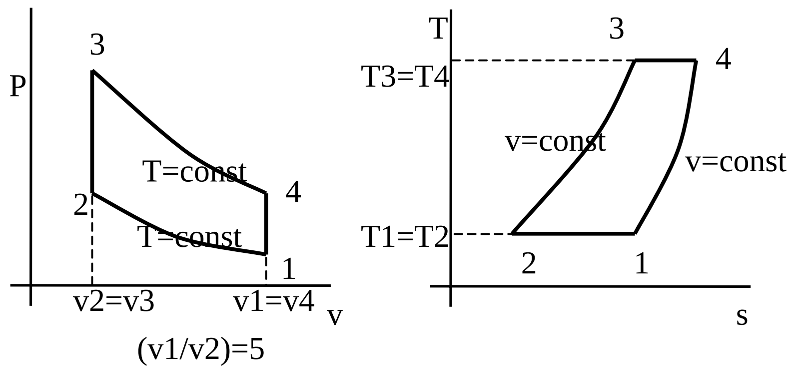
Where $u_1 = u_{f1} + u_{fg1} = 417.33 + (0.8)2088.72 = 2088.306$ kJ/kg

$u_2 = u_{g,800kPa} = 2576.79$ kJ/kg

Therefore, $W_{1-2} = m(u_1 - u_2) = 3(208.306 - 2576.79) = -1465.45$ kJ (work done on the system) ------------©

3.

Given: Air in Piston-Cylinder assembly undergoes an ideal Stirling cycle (P-V and T-S diagrams)



Air is treated as an *ideal gas*

State 1: P_1 = 100kPa, T_1 =30^{o}C (=303.15 K)

State 3: T_3 = 1200^{o}C (=1473.15 K)

From the given processes,

$T_2 = T_1$ =303.15 K;

$T_3 = T_4$ = 1473.15 K;

$v_2 = v_3$;

$v_4 = v_1$;

$$\frac{v_1}{v_2} = \frac{v_4}{v_3} = 5$$

1-2: isothermal process; $P_1 v_1 = P_2 v_2 \Rightarrow P_2 = P_1\left(\frac{v_1}{v_2}\right) = 100(5) = 500$ kPa --------------------------------(a)

2-3: const. volume process; $\frac{P_2}{T_2} = \frac{P_3}{T_3} \Rightarrow P_3 = P_2\left(\frac{T_3}{T_2}\right) = 500\left(\frac{1473.15}{303.15}\right) = 2429.73$ kPa -------------(a)

3-4: isothermal process; $P_3 v_3 = P_4 v_4 \Rightarrow P_4 = P_3\left(\frac{v_3}{v_4}\right) = 2429.73(\frac{1}{5}) = 485.95$ kPa -------------------(a)

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287)(303.15)}{100} = 0.87 \text{ m}^3\text{/kg}$$

Therefore, v_4 = 0.87 m³/kg; $v_2 = v_3 = \frac{v_1}{5}$ = 0.174 m³/kg

1-2: isothermal process; $\therefore w_{1-2} = P_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = (100)(0.87)\ln\left(\frac{1}{5}\right) = -140.02$ kJ/kg ----------------(b)

3-4: isothermal process; $\therefore w_{3-4} = P_3 v_3 \ln\left(\frac{v_4}{v_3}\right) = (2429.73)(0.174)\ln 5 = 680.45$ kJ/kg ------------(b)

FLT for control mass for a process *a-b*: ${}_a\Delta u_b = q_{a-b} - w_{a-b}$

1-2: isothermal process; $\therefore {}_1\Delta u_2 = 0 \quad q_{1-2} = w_{1-2} = -140.02$ kJ/kg -------------------------------------(c)

2-3: const. volume process;

$\therefore w_{2-3} = 0 \quad q_{2-3} = {}_2\Delta u_3 = C_v(T_3 - T_2) = (0.717)(1473.15 - 303.15) = 838.89$ kJ/kg --------(c)

3-4: isothermal process; $\therefore {}_3\Delta u_4 = 0 \quad q_{3-4} = w_{3-4} = 680.45$ kJ/kg ---------------------------------------(c)

4-1: const. volume process;

$\therefore w_{4-1} = 0 \quad q_{4-1} = {}_4\Delta u_1 = C_v(T_1 - T_4) = (0.717)(303.15 - 1473.15) = -838.89$ kJ/kg --------(c)

Thermal Efficiency: $\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{w_{1-2} + w_{3-4}}{q_{3-4}} = \frac{-140.02 + 680.45}{680.45} = 0.7942$

$\eta_{th} = 79.42\%$ --(d)

4.

Given: Rankine cycle; water is the working substance (Use Tables B.1.1 to B.1.4)

State 1: Saturated liquid at P_1 = 10 kPa
State 3: P_3 = 5 MPa; T_3 = 500°C (=773.15 K)
State 4: P_4 = 200 kPa
State 5: T_5 = 300°C (=573.15 K)

Boilers & condenser operate at const. P: $P_2 = P_3$ = 5 Mpa; $P_5 = P_4$ = 200 kPa; $P_6 = P_1$ = 10 kPa

Each turbine stage has an isentropic efficiency of 85%. *i.e.,*

$w_{3-4} = 0.85 w_{3-4s}; w_{5-6} = 0.85 w_{5-6s}$

From State 3: h_3 = 3433.76 kJ/kg; s_3 = 6.9758 kJ/kgK
For 3-4s; $s_{4s} = s_3$ = 6.9758 kJ/kgK
At P_4, $s_{f4} < s_{4s} < s_{fg4}$. Therefore State 4s is saturated.

$x_{4s} = \frac{s_{4s} - s_{f4}}{s_{fg4}} = \frac{6.9758 - 1.53}{5.59} = 0.974$

$h_{4s} = h_{f4} + x_{4s} h_{fg4} = 504.68 + (0.974)2201.96 = 2649.39$ kJ/kg

From FLT for CV: $w_{3-4s} = h_3 - h_{4s} = 3433.76 - 2649.39 = 784.37$ kJ/kg

Therfore, $w_{3-4} = 0.85 w_{3-4s} = (0.85)784.37 = 666.71$ kJ/kg ------------------------------------(a)

From State 5: h_5 = 3071.79 kJ/kg; s_3 = 7.8926 kJ/kgK
For 5-6s; $s_{6s} = s_5$ = 7.8926 kJ/kgK
At P_6, $s_{f6} < s_{6s} < s_{fg6}$. Therefore State 6s is saturated.

$$x_{6s}=\frac{s_{6s}-s_{f6}}{s_{fg6}}=\frac{7.8926-0.6492}{7.5010}=0.9656$$

$h_{6s}=h_{f6}+x_{6s}h_{fg6}=191.81+(0.9656)2392.82=2502.31$ kJ/kg

From FLT for CV: $w_{5-6s}=h_5-h_{6s}=3071.79-2502.31=569.47$ kJ/kg

Therfore, $w_{5-6}=0.85w_{5-6s}=(0.85)569.47=484.05$ kJ/kg ------------------------------------(a)

$v_1=v_{f,10kPa}=0.001\,\text{m}^3/\text{kg}$

Boiler 1: From FLT for CV: $q_{2-3}=h_3-h_2$

From FLT for CV for pump, 1-2: $w_{1-2}=h_1-h_2$ $\quad h_2=h_1-w_{1-2}$

$w_{1-2}=-v_1(P_2-P_1)=-0.001(5000-10)=-4.99$ kJ/kg

$\therefore h_2=h_1-w_{1-2}=191.81-(-4.99)=196.8$ kJ/kg

$\therefore q_{2-3}=h_3-h_2=3433.76-196.8=3236.96$ kJ/kg --(b)

Boiler 2: From FLT for CV: $q_{5-4}=h_5-h_4$

From FLT for CV: $w_{3-4}=h_3-h_4$ $\quad h_4=h_3-w_{3-4}=3433.76-666.71=2767.05$ kJ/kg

$\therefore q_{5-4}=h_5-h_4=3071.79-2767.05=304.74$ kJ/kg --(b)

Work done on the pump is $w_p=w_{1-2}=-4.99$ kJ/kg ---(c)

Thermal Efficiency: $$\eta_{th}=\frac{w_{net}}{q_{in}}=\frac{w_{3-4}+w_{5-6}+w_{1-2}}{q_{2-3}+q_{4-5}}=\frac{661.71+484.05-4.99}{3236.96+304.74}=0.322$$

$\eta_{th}=32.2\%$ --(d)