ME 24-221
THERMODYNAMICS – I

Solutions to extra problem set from Chapters 5, 6 and 7.
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J. Y. Murthy

5.61 Saturated, x = 1%, water at 25°C is contained in a hollow spherical aluminum vessel with inside diameter of 0.5 m and a 1-cm thick wall. The vessel is heated until the water inside is saturated vapor. Considering the vessel and water together as a control mass, calculate the heat transfer for the process.

C.V. Vessel and water. This is a control mass of constant volume.

$m_2 = m_1$; $U_2 - U_1 = {}_1Q_2 - {}_1W_2 = {}_1Q_2$

State 1: $v_1 = 0.001003 + 0.01 \times 43.359 = 0.4346 \text{ m}^3/\text{kg}$

$u_1 = 104.88 + 0.01 \times 2304.9 = 127.9 \text{ kJ/kg}$

State 2: $x_2 = 1$ and constant volume so $v_2 = v_1 = V/m$

$v_{g\,T2} = v_1 = 0.4346 \Rightarrow T_2 = 146.1°C$; $u_2 = u_{G2} = 2555.9$

$$V_{INSIDE} = \frac{\pi}{6}(0.5)^3 = 0.06545 \text{ m}^3 \text{ ; } m_{H_2O} = \frac{0.06545}{0.4346} = 0.1506 \text{ kg}$$

$$V_{Al} = \frac{\pi}{6}((0.52)^3 - (0.5)^3) = 0.00817 \text{ m}^3$$

$m_{Al} = \rho_{Al}V_{Al} = 2700 \times 0.00817 = 22.065 \text{ kg}$

${}_1Q_2 = U_2 - U_1 = m_{H_2O}(u_2 - u_1)_{H_2O} + m_{Al}C_{V\,Al}(T_2 - T_1)$

$= 0.1506(2555.9 - 127.9) + 22.065 \times 0.9(146.1 - 25)$

= **2770.6 kJ**

5.63 A rigid insulated tank is separated into two rooms by a stiff plate. Room A of 0.5 m^3 contains air at 250 kPa, 300 K and room B of 1 m^3 has air at 150 kPa, 1000 K. The plate is removed and the air comes to a uniform state without any heat transfer. Find the final pressure and temperature.

C.V. Total tank. Control mass of constant volume.

Mass and volume: $m_2 = m_A + m_B$; $V = V_A + V_B = 1.5 \text{ m}^3$

Energy Eq.: $m_2 u_2 - m_A u_{A1} - m_B u_{B1} = Q - W = 0$

Ideal gas at 1: $m_A = P_{A1}V_A/RT_{A1} = 250 \times 0.5/(0.287 \times 300) = 1.452 \text{ kg}$

$u_{A1} = 214.364$ kJ/kg from Table A.7

Ideal gas at 2: $m_B = P_{B1}V_B/RT_{B1} = 150 \times 1/(0.287 \times 1000) = 0.523 \text{ kg}$

$u_{B1} = 759.189$ kJ/kg from Table A.7

$m_2 = m_A + m_B = 1.975 \text{ kg}$

$u_2 = (m_A u_{A1} + m_B u_{B1})/m_2 = (1.452 \times 214.364 + 0.523 \times 759.189)/1.975$

$= 358.64$ kJ/kg => Table A.7 $\mathbf{T_2 = 498.4 \text{ K}}$

$P_2 = m_2 RT_2/V = 1.975 \times 0.287 \times 498.4/1.5 =$ **188.3 kPa**

5.71 Two containers are filled with air, one a rigid tank A, and the other a piston/cylinder B that is connected to A by a line and valve, as shown in Fig. P5.71. The initial conditions are: $m_A = 2$ kg, $T_A = 600$ K, $P_A = 500$ kPa and $V_B = 0.5$ m^3, $T_B = 27$°C, $P_B = 200$ kPa. The piston in B is loaded with the outside atmosphere and the piston mass in the standard gravitational field. The valve is now opened, and the air comes to a uniform condition in both volumes. Assuming no heat transfer, find the initial mass in B, the volume of tank A, the final pressure and temperature and the work, $_1W_2$.

Cont.: $m_2 = m_1 = m_{A1} + m_{B1}$

Energy: $m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = -{}_1W_2$; $\quad {}_1W_2 = P_{B1}(V_2 - V_1)$

System: $P_B = \text{const} = P_{B1} = P_2$; Substance: $PV = mRT$

$m_{B1} = P_{B1}V_{B1}/RT_{B1} =$ **1.161 kg** ; $V_A = m_{A1}RT_{A1}/P_{A1} =$ **0.6888 m³**

$P_2 = P_{B1} =$ **200 kPa** ; A.7: $u_{A1} = 434.8$, $u_{B1} = 214.09$ kJ/kg

$m_2u_2 + P_2V_2 = m_{A1}u_{A1} + m_{B1}u_{B1} + P_{B1}V_1 = m_2h_2 = 1355.92$ kJ

$\Rightarrow h_2 = 428.95$ kJ/kg $\Rightarrow$ $T_2 = 427.7$ K $\Rightarrow$ $V_2 = m_{tot}RT_2/P_2 = 1.94$ m^3

$_1W_2 = 200 \times (1.94 - 1.1888) =$ **150.25 kJ**

5.83 Water at 150°C, quality 50% is contained in a cylinder/piston arrangement with initial volume 0.05 m^3. The loading of the piston is such that the inside pressure is linear with the square root of volume as $P = 100 + CV^{0.5}$ kPa. Now heat is transferred to the cylinder to a final pressure of 600 kPa. Find the heat transfer in the process.

Continuty: $m_2 = m_1$ Energy: $m(u_2 - u_1) = {}_1Q_2 - {}_1W_2$

State 1: $v_1 = 0.1969$, $u_1 = 1595.6$ kJ/kg $\Rightarrow$ $m = V/v_1 = 0.254$ kg

Process equation $\Rightarrow$ $P_1 - 100 = CV_1^{1/2}$ so

$(V_2/V_1)^{1/2} = (P_2 - 100)/(P_1 - 100)$

$$V_2 = V_1 \text{ x} \left[\frac{P_2 - 100}{P_1 - 100}\right]^2 = 0.05 \text{ x} \left[\frac{500}{475.8 - 100}\right]^2 = 0.0885$$

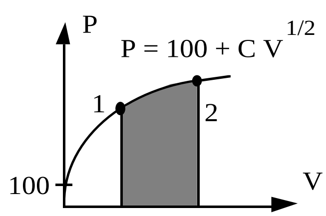
$$_1W_2 = \int PdV = \int (100 + CV^{1/2})dV = 100\text{x}(V_2 - V_1) + \frac{2}{3}C(V_2^{1.5} - V_1^{1.5})$$

$$= 100(V_2 - V_1)(1 - 2/3) + (2/3)(P_2V_2 - P_1V_1)$$

$_1W_2 = 100\ (0.0885\text{-}0.05)/3 + 2\ (600 \text{ x } 0.0885\text{-}475.8 \text{ x } 0.05)/3 = 20.82$ kJ

State 2: P_2, $v_2 = V_2/m = 0.3484$ $\Rightarrow$ $u_2 = 2631.9$ kJ/kg, $T_2 \cong 196$°C

$_1Q_2 = 0.254 \text{ x } (2631.9 - 1595.6) + 20.82 =$ **284 kJ**



5.85 A closed cylinder is divided into two rooms by a frictionless piston held in place by a pin, as shown in Fig. P5.85. Room A has 10 L air at 100 kPa, 30°C, and room B has 300 L saturated water vapor at 30°C. The pin is pulled, releasing the piston, and both rooms come to equilibrium at 30°C and as the water is compressed it becomes two-phase. Considering a control mass of the air and water, determine the work done by the system and the heat transfer to the cylinder.

$P_2 = P_{G\ H_2O\ at\ 30°C} = P_{A2} = P_{B2} = 4.246$ kPa

Air, I.G.: $P_{A1}V_{A1} = m_A R_A T = P_{A2}V_{A2} = P_{G\ H_2O\ at\ 30°C}V_{A2}$

$\rightarrow V_{A2} = \frac{100 \text{ x } 0.01}{4.246} \text{ m}^3 = 0.2355 \text{ m}^3$

$V_{B2} = V_{A1} + V_{B1} - V_{A2} = 0.30 + 0.01 - 0.2355 = 0.0745 \text{ m}^3$

$m_B = \frac{V_{B1}}{v_{B1}} = \frac{0.3}{32.89} = 9.121\text{x}10^{-3}$ kg => $v_{B2} = 8.166 \text{ m}^3\text{/kg}$

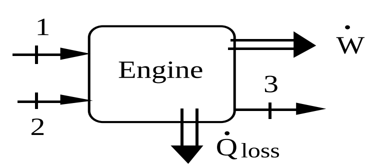
$8.166 = 0.001004 + x_{B2} \text{ x } (32.89 - 0.001) \Rightarrow x_{B2} = 0.2483$

System A+B: W = 0; $\Delta U_A = 0$ (IG & $\Delta T = 0$)

$u_{B2} = 125.78 + 0.2483 \text{ x } 2290.8 = 694.5$, $u_{B1} = 2416.6$ kJ/kg

${}_1Q_2 = 9.121\text{x}10^{-3}(694.5 - 2416.6) =$ **-15.7 kJ**

6.34 A large SSSF expansion engine has two low velocity flows of water entering. High pressure steam enters at point 1 with 2.0 kg/s at 2 MPa, 500°C and 0.5 kg/s cooling water at 120 kPa, 30°C enters at point 2. A single flow exits at point 3 with 150 kPa, 80% quality, through a 0.15 m diameter exhaust pipe. There is a heat loss of 300 kW. Find the exhaust velocity and the power output of the engine.



C.V. : Engine (SSSF)

Constant rates of flow, $\dot{Q}_{loss}$ and $\dot{W}$

State 1: Table B.1.3: $h_1 = 3467.6$

State 2: Table B.1.1: $h_2 = 125.77$

$h_3 = 467.1 + 0.8 \times 2226.5 = 2248.3$ kJ/kg

$v_3 = 0.00105 + 0.8 \times 1.15825 = 0.92765 \text{ m}^3\text{/kg}$

Continuity : $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 2 + 0.5 = 2.5$ kg/s $= (AV/v) = (\pi/4)D^2V/v$

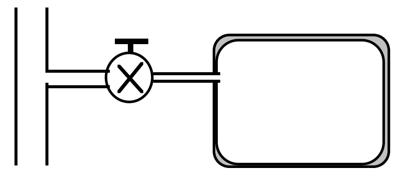
Energy Eq. : $\dot{m}_1h_1 + \dot{m}_2h_2 = \dot{m}_3(h_3 + 0.5\ V^2) + \dot{Q}_{loss} + \dot{W}$

$\mathbf{V} = \dot{m}_3v_3 / [(\pi/4)D^2] = 2.5 \times 0.92765/(0.7854 \times 0.15^2) =$ **131.2 m/s**

$0.5\ \mathbf{V}^2 = 0.5 \times 131.2 \times 131.2/1000 = 8.6$ kJ/kg (remember units factor 1000)

$\dot{W} = 2 \times 3467.6 + 0.5 \times 125.77 - 2.5\ (2248.3 + 8.6) - 300 =$ **1056 kW**

6.49 A 25-L tank, shown in Fig. P6.49, that is initially evacuated is connected by a valve to an air supply line flowing air at 20°C, 800 kPa. The valve is opened, and air flows into the tank until the pressure reaches 600 kPa.Determine the final temperature and mass inside the tank, assuming the process is adiabatic. Develop an expression for the relation between the line temperature and the final temperature using constant specific heats.



a) C.V. Tank: Continuity Eq.: $m_i = m_2$

Energy Eq.: $m_i h_i = m_2 u_2$

$u_2 = h_i = 293.64$ (Table A.7)

$\Rightarrow T_2 =$ **410.0 K**

$$m_2 = \frac{P_2 V}{RT_2} = \frac{600 \text{ x } 0.025}{0.287 \text{ x } 410} = \mathbf{0.1275\ kg}$$

b) Assuming constant specific heat,

$$h_i = u_i + RT_i = u_2\ , \quad RT_i = u_2 - u_i = C_{Vo}(T_2 - T_i)$$

$$C_{Vo}T_2 = (\ C_{Vo} + R\ )T_i = C_{Po}T_i\ , \quad T_2 = \left(\frac{C_{Po}}{C_{Vo}}\right)T_i = kT_i$$

For $T_i = 293.2\text{K}$ & constant C_{Po}, $T_2 = 1.40\text{x}293.2 =$ **410.5K**

6.59 A 750-L rigid tank, shown in Fig. P6.59, initially contains water at 250°C, 50% liquid and 50% vapor, by volume. A valve at the bottom of the tank is opened, and liquid is slowly withdrawn. Heat transfer takes place such that the temperature remains constant. Find the amount of heat transfer required to the state where half the initial mass is withdrawn.

CV: vessel

$$m_{LIQ1} = \frac{0.375}{0.001251} = 299.76 \text{ kg}; \quad m_{VAP1} = \frac{0.375}{0.05013} = 7.48 \text{ kg}$$

$m_1 = 307.24$ kg; $m_e = m_2 = 153.62$ kg

$$v_2 = \frac{0.75}{153.62} = 0.004882 = 0.001251 + x_2 \text{ x } 0.04888$$

$x_2 = 0.07428$; $u_2 = 1080.39 + 0.07428 \text{ x } 1522 = 1193.45$

$m_1 u_1 = 299.76 \text{ x } 1080.39 + 7.48 \text{ x } 2602.4 = 343324$ kJ

$Q_{CV} = m_2 u_2 - m_1 u_1 + m_e h_e$

$= 153.62 \text{ x } 1193.45 - 343324 + 153.62 \text{ x } 1085.36 =$ **6744 kJ**

6.62 An insulated spring-loaded piston/cylinder, shown in Fig. P6.62, is connected to an air line flowing air at 600 kPa, 700 K by a valve. Initially the cylinder is empty and the spring force is zero. The valve is then opened until the cylinder pressure reaches 300 kPa. By noting that $u_2 = u_{line} + C_V(T_2 - T_{line})$ and $h_{line} - u_{line} = RT_{line}$ find an expression for T_2 as a function of P_2, P_o, T_{line}. With P = 100 kPa, find T_2.

C.V. Air in cylinder, insulated so $_1Q_2 = 0$

Cont.: $m_2 - m_1 = m_{in}$; Energy Eq.: $m_2u_2 - m_1u_1 = m_{in}h_{line} - {}_1W_2$

$$m_1 = 0 \Rightarrow m_{in} = m_2 ;\ m_2u_2 = m_2h_{line} - \frac{1}{2}(P_0 + P_2)m_2v_2$$

$$\Rightarrow \quad u_2 + \frac{1}{2}(P_0 + P_2)v_2 = h_{line}$$

$$C_V(T_2 - T_{line}) + u_{line} + \frac{1}{2}(P_0 + P_2)RT_2/P_2 = h_{line}$$

$$\mathbf{\left[C_V + \frac{1}{2}\frac{P_0 + P_2}{P_2}R\right]T_2 = (R + C_V)T_{line}}$$

$$\text{with \#'s: } T_2 = \frac{R + C_V}{\frac{2}{3}R + C_V}T_{line} ;\quad C_V/R = 1/(k-1) ,\quad k = 1.4$$

$$T_2 = \frac{k - 1 + 1}{\frac{2}{3}k - \frac{2}{3} + 1}T_{line} = \frac{3k}{2k + 1}T_{line} = 1.105\ T_{line} = \mathbf{773.7\ K}$$

6.66 A spherical balloon is constructed of a material such that the pressure inside is proportional to the balloon diameter to the power 1.5. The balloon contains argon gas at 1200 kPa, 700°C, at a diameter of 2.0 m. A valve is now opened, allowing gas to flow out until the diameter reaches 1.8 m, at which point the temperature inside is 600°C. The balloon then continues to cool until the diameter is 1.4 m.

a) How much mass was lost from the balloon?
b) What is the final temperature inside?
c) Calculate the heat transferred from the balloon during the overall process.

C.V. Balloon. Process 1 - 2 - 3. Flow out in 1- 2, USUF.

Process: $P \propto D^{3/2}$ and since $V \propto D^3$ => $P = C\ V^{1/2}$

State 1: $T_1 = 700^oC$, $P_1 = 1200$ kPa, $V_1 = (\pi/6)\ D_1^3 = 4.188\ m^3$

$$m_1 = P_1V_1/RT_1 = 1200x4.1888/(0.20813x973.15) = 24.816\ kg$$

State 2: $T_2 = 600^oC$, $V_2 = (\pi/6)\ D_2^3 = 3.0536\ m^3$

$$P_2 = P_1\ (V_2/V_1)^{1/2} = 1200\ (3.0536/4.1888)^{1/2} = 1025\ kPa$$

$$m_3 = m_2 = P_2V_2/RT_2 = 1025x3.0536/(0.20813x873.15) = 17.222\ kg$$

a) $m_E = m_1 - m_2 =$ **7.594 kg**

State 3: $D_3 = 1.4$ m => $V_3 = (\pi/6)\ D_3^3 = 1.4368\ m^3$

$$P_3 = 1200\ (1.4368/4.1888)^{1/2} = 703\ kPa$$

b) $T_3 = P_3V_3/m_3R = 703x1.4368/(17.222x0.20813) =$ **281.8 K**

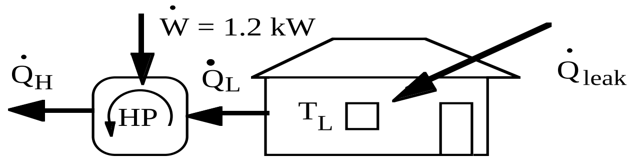
c) Process is polytropic with n = -1/2 so the work becomes

$${}_1W_3 = \int P\,dV = \frac{P_3V_3 - P_1V_1}{1 - n} = \frac{703x1.4368 - 1200x4.1888}{1 - (-0.5)} = -2677.7\ kJ$$

$${}_1Q_3 = m_3\,u_3 - m_1u_1 + m_eh_e + {}_1W_3$$
$$= 17.222x0.312x281.8 - 24.816x0.312x973.15$$
$$+ 7.594x0.52x(973.15+873.15)/2 - 2677.7$$
$$= 1515.2 - 7539.9 + 3647.7 - 2677.7 = \mathbf{-5054.7\ kJ}$$

7.20 A heat pump cools a house at 20°C with a maximum of 1.2 kW power input. The house gains 0.6 kW per degree temperature difference to the ambient and the heat pump coefficient of performance is 60% of the theoretical maximum. Find the maximum outside temperature for which the heat pump provides sufficient cooling.

Solution:



Here:
$T_L = T_{house}$
$T_H = T_{amb}$

In this setup the low temperature space is the house and the high temperature space is the ambient. The heat pump must remove the gain or leak heat transfer to keep it at a constant temperature.

$\dot{Q}_{leak} = 0.6\ (T_{amb} - T_{house}) = \dot{Q}_L$ which must be removed by the heat pump.

$$\beta' = \dot{Q}_H / \dot{W} = 1 + \dot{Q}_L / \dot{W} = 0.6\ \beta'_{carnot} = 0.6\ T_{amb} / (T_{amb} - T_{house})$$

Substitute in for $\dot{Q}_L$ and multiply with $(T_{amb} - T_{house})$:

$$(T_{amb} - T_{house}) + 0.6\ (T_{amb} - T_{house})^2 / \dot{W} = 0.6\ T_{amb}$$

Since $T_{house} = 293.15$ K and $\dot{W} = 1.2$ kW it follows

$$T_{amb}^2 - 585.5\ T_{amb} + 85350.6 = 0$$

Solving = > T_{amb} = **311.51 K = 38.36 °C**

7.30 An air-conditioner with a power input of 1.2 kW is working as a refrigerator (β = 3) or as a heat pump (β' = 4). It maintains an office at 20°C year round which exchanges 0.5 kW per degree temperature difference with the atmosphere. Find the maximum and minimum outside temperature for which this unit is sufficient.

Solution:

Analyse the unit in heat pump mode

Replacement heat transfer equals the loss: $\dot{Q} = 0.5\ (T_H - T_{amb})$

$$\dot{W} = \dot{Q}_H / \beta' = 0.5\ (T_H - T_{amb}) / 4$$

$$T_H - T_{amb} = 4\ \dot{W} / 0.5 = 9.6$$

Heat pump mode: Minumum T_{amb} = 20 - 9.6 = **10.4 °C**

The unit as a refrigerator must cool with rate: $\dot{Q} = 0.5\ (T_{amb} - T_{house})$

$$\dot{W} = \dot{Q}_L / \beta = 0.5\ (T_{amb} - T_{house}) / 3$$

$$T_{amb} - T_{house} = 3\ \dot{W} / 0.5 = 7.2$$

Refrigerator mode: Maximum T_{amb} = 20 + 7.2 = **27.2 °C**

7.42 A furnace, shown in Fig. P7.42, can deliver heat, Q_{H1} at T_{H1} and it is proposed to use this to drive a heat engine with a rejection at T_{atm} instead of direct room heating. The heat engine drives a heat pump that delivers Q_{H2} at T_{room} using the atmosphere as the cold reservoir. Find the ratio Q_{H2}/Q_{H1} as a function of the temperatures. Is this a better set-up than direct room heating from the furnace?

Solution:

C.V.: Heat Eng.: $\dot{W}_{HE} = \eta\dot{Q}_{H1}$ where $\eta = 1 - T_{atm}/T_{H1}$

C.V.: Heat Pump: $\dot{W}_{HP} = \dot{Q}_{H2}/\beta'$ where $\beta' = T_{rm}/(T_{rm} - T_{atm})$

Work from heat engine goes into heat pump so we have

$$\dot{Q}_{H2} = \beta' \dot{W}_{HP} = \beta' \eta \dot{Q}_{H1}$$

and we may substitute T's for β', η. If furnace is used directly $\dot{Q}_{H2} = \dot{Q}_{H1}$, so if $\beta'\eta > 1$ this proposed setup is better. Is it? For $T_{H1} > T_{atm}$ formula shows that it is good for Carnot cycles. In actual devices it depends wether $\beta'\eta > 1$ is obtained.

7.44 In a cryogenic experiment you need to keep a container at −125°C although it gains 100 W due to heat transfer. What is the smallest motor you would need for a heat pump absorbing heat from the container and rejecting heat to the room at 20°C?

Solution:

$$\beta'_{HP} = \dot{Q}_H / \dot{W} = \frac{T_H}{T_H - T_L} = \frac{293.15}{20 - (-125)} = 2.022 = 1 + \dot{Q}_L / \dot{W}$$

=> $\dot{W} = \dot{Q}_L/(\beta' - 1) = 100/1.022 =$ **97.8 W**

7.51 Air in a piston/cylinder goes through a Carnot cycle with the P-v diagram shown in Fig. 7.24. The high and low temperatures are 600 K and 300 K respectively. The heat added at the high temperature is 250 kJ/kg and the lowest pressure in the cycle is 75 kPa. Find the specific volume and pressure at all 4 states in the cycle assuming constant specific heats at 300 K..

Solution:

q_H = 250 kJ/kg , T_H = 600 K, T_L = 300 K, P_3 = 75 kPa

C_v = 0.717 ; R = 0.287

1: 600 K , 2: 600 K, 3: 75 kPa, 300 K 4: 300 K

$v_3 = RT_3 / P_3 = 0.287 \times 300 / 75 = 1.148 \text{ m}^3/\text{kg}$

2→3 Eq.7.11 & C_v = const $\Rightarrow C_v \ln (T_L / T_H) + R \ln (v_3/v_2) = 0$

$\Rightarrow \ln (v_3/v_2) = - (C_v / R) \ln (T_L / T_H)$

$= - (0.7165/0.287) \ln (300/600) = 1.73045$

$\Rightarrow v_2 = v_3 / \exp (1.73045) = 1.148/5.6432 = 0.2034 \text{ m}^3/\text{kg}$

1→2 $q_H = RT_H \ln (v_2 / v_1)$

$\ln (v_2 / v_1) = q_H / RT_H = 250/0.287 \times 600 = 1.4518$

$v_1 = v_2 / \exp (1.4518) = 0.04763 \text{ m}^3/\text{kg}$

$v_4 = v_1 \times v_3 / v_2 = 0.04763 \times 1.148/0.2034 = 0.2688$

$P_1 = RT_1 / v_1 = 0.287 \times 600/0.04763 = 3615 \text{ kPa}$

$P_2 = RT_2 / v_2 = 0.287 \times 600/0.2034 = 846.6 \text{ kPa}$

$P_4 = RT_4 / v_4 = 0.287 \times 300/0.2688 = 320 \text{ kPa}$