ME 24-221
THERMODYNAMICS I

Solutions to extra problems in Chapters 2, 3 and 4:
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**2.25** Two reservoirs, A and B, open to the atmosphere, are connected with a mercury manometer. Reservoir A is moved up/down so the two top surfaces are level at $h_3$ as shown in Fig. P2.25. Assuming that you know $\rho_A$, $\rho_{Hg}$ and measure the heights $h_1$, $h_2$ , and $h_3$, find the density $\rho_B$.

Solution:

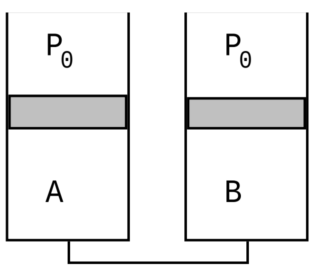
Balance forces on each side:

$$P_0 + \rho_A g(h_3 - h_2) + \rho_{Hg} g h_2 = P_0 + \rho_B g(h_3 - h_1) + \rho_{Hg} g h_1$$

$$\Rightarrow \quad \rho_{\mathbf{B}} = \rho_{\mathbf{A}}\left(\frac{\mathbf{h_3 - h_2}}{\mathbf{h_3 - h_1}}\right) + \rho_{\mathbf{Hg}}\left(\frac{\mathbf{h_2 - h_1}}{\mathbf{h_3 - h_1}}\right)$$

**2.32** Two piston/cylinder arrangements, A and B, have their gas chambers connected by a pipe. Cross-sectional areas are $A_A = 75\ cm^2$ and $A_B = 25\ cm^2$ with the piston mass in A being $m_A = 25$ kg. Outside pressure is 100 kPa and standard gravitation. Find the mass $m_B$ so that none of the pistons have to rest on the bottom.

Solution:




Force balance for both pistons: $F\uparrow = F\downarrow$

A: $m_{PA} g + P_0 A_A = P A_A$

B: $m_{PB} g + P_0 A_B = P A_B$

Same P in A and B gives no flow between them.

$$\frac{m_{PA} g}{A_A} + P_0 = \frac{m_{PB} g}{A_B} + P_0$$

=> $m_{PB} = m_{PA}\ A_A /\ A_B = 25 \times 25/75 =$ **8.33 kg**

**2.36** Two cylinders are connected by a piston as shown in Fig. P2.36. Cylinder A is used as a hydraulic lift and pumped up to 500 kPa. The piston mass is 25 kg and there is standard gravity. What is the gas pressure in cylinder B?

Solution:

Force balance for the piston: $P_B A_B + m_p g + P_0(A_A - A_B) = P_A A_A$

$A_A = (\pi/4)0.1^2 = 0.00785 \text{ m}^2$; $A_B = (\pi/4)0.025^2 = 0.000491 \text{ m}^2$

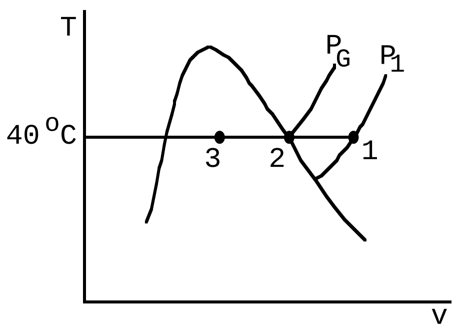
$P_B A_B = P_A A_A - m_p g - P_0(A_A - A_B) = 500\times 0.00785 - (25 \times 9.807/1000)$

$- 100\ (0.00785 - 0.000491) = 2.944 \text{ kN}$

$P_B = 2.944/0.000491 = 5996 \text{ kPa} = \mathbf{6.0\ MPa}$

**4.21** A cylinder having an initial volume of 3 $m^3$ contains 0.1 kg of water at 40°C. The water is then compressed in an isothermal quasi-equilibrium process until it has a quality of 50%. Calculate the work done in the process. Assume the water vapor is an ideal gas.

Solution: C.V. Water




$v_1 = V_1/m = \frac{3}{0.1} = 30 \text{ m}^3/\text{kg} ( > v_G )$

Tbl B.1.1 => $P_G$ = 7.384 kPa very low

so $H_2O$ ~ ideal gas from 1-2

$$P_1 = P_G \frac{v_G}{v_1} = 7.384 \times \frac{19.52}{30} = 4.8 \text{ kPa}$$

$V_2 = mv_2 = 0.1 \times 19.52 = 1.952 \text{ m}^3$

$$\text{T = C:} \quad W_{12} = \int_1^2 PdV = P_1V_1 \ln\frac{V_2}{V_1} = 4.8 \times 3.0 \times \ln\frac{1.952}{3} = -6.19 \text{ kJ}$$

$v_3 = 0.001008 + 0.5 \times 19.519 = 9.7605 \quad => \quad V_3 = mv_3 = 0.976 \text{ m}^3$

$$P = C = P_g: \quad W_{23} = \int_2^3 PdV = P_g\ (V_3-V_2) = 7.384(0.976 - 1.952) = -7.21 \text{ kJ}$$

Total work: $W_{13} = -6.19 - 7.21 = \mathbf{-13.4\ kJ}$

**3.12** Air in a tank is at 1 MPa and room temperature of 20°C. It is used to fill an initially empty balloon to a pressure of 200 kPa, at which point the diameter is 2 m and the temperature is 20°C. Assume the pressure in the balloon is linearly proportional to its diameter and that the air in the tank also remains at 20°C throughout the process. Find the mass of air in the balloon and the minimum required volume of the tank.

Solution: Assume air is an ideal gas.

Balloon final state: $V_2 = (4/3)\,\pi\, r^3 = (4/3)\,\pi\, 2^3 = 33.51\ m^3$

$m_{2bal} = P_2\, V_2 / RT_2 = 200\times 33.51 / 0.287 \times 293.15 =$ **79.66 kg**

Tank must have $P_2 \ \geq\ 200$ kPa $\Rightarrow$ $m_{2\ tank} \geq\ P_2\, V_{TANK} / RT_2$

Initial mass must be enough: $m_1 = m_{2bal} + m_{2\ tank} = P_1 V_1 / R\, T_1$

$P_1 V_{TANK} / R\, T_1 = m_{2bal} + P_2 V_{TANK} / RT_2 \quad \Rightarrow$

$V_{TANK} = RTm_{2bal} / (P_1 - P_2) = 0.287 \times 293.15 \times 79.66 / (1000 - 200)$

$=$ **8.377 m³**

**3.22** Is it reasonable to assume that at the given states the substance behaves as an ideal gas?

Solution:

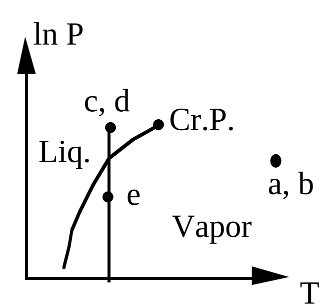
a) Oxygen, $O_2$ at 30°C, 3 MPa **Ideal Gas** ( $T \gg T_c = 155$ K from A.2)

b) Methane, $CH_4$ at 30°C, 3 MPa **Ideal Gas** ( $T \gg T_c = 190$ K from A.2)

c) Water, $H_2O$ at 30°C, 3 MPa **NO** compressed liquid $P > P_{sat}$ (B.1.1)

d) R-134a at 30°C, 3 MPa **NO** compressed liquid $P > P_{sat}$ (B.5.1)

e) R-134a at 30°C, 100 kPa **Ideal Gas** P is low $< P_{sat}$ (B.5.1)



**3.35** Determine the mass of methane gas stored in a 2 $m^3$ tank at –30°C, 3 MPa. Estimate the percent error in the mass determination if the ideal gas model is used.

Solution:

The methane Table B.7.2 linear interpolation between 225 and 250 K.

$$\Rightarrow \quad v \cong 0.03333 + \frac{243.15\text{-}225}{250\text{-}225} \times (0.03896\text{-}0.03333) = 0.03742 \text{ m}^3\text{/kg}$$

$m = V/v = 2/0.03742 =$ **53.45 kg**

Ideal gas assumption

$v = RT/P = 0.51835 \times 243.15/3000 = 0.042$

$m = V/v = 2/0.042 = 47.62$ kg

Error: 5.83 kg **10.9% too small**

**4.10** A piston/cylinder contains 1 kg of liquid water at 20°C and 300 kPa. Initially the piston floats, similar to the setup in Problem 4.7, with a maximum enclosed volume of 0.002 $m^3$ if the piston touches the stops. Now heat is added so a final pressure of 600 kPa is reached. Find the final volume and the work in the process.

Solution:

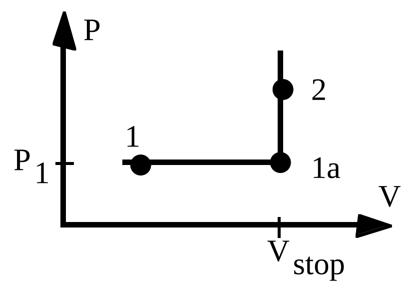
Take CV as the water which is a control mass: $m_2 = m_1 = m$ ;

Table B.1.1: 20°C => $P_{sat}$ = 2.34 kPa

State 1: Compressed liquid

$v = v_f(20) = 0.001002 \text{ m}^3\text{/kg}$

State 1a: $v_{stop} = 0.002 \text{ m}^3\text{/kg}$ , 300 kPa




State 2: Since P = 600 > $P_{lift}$ then $v = v_{stop} = 0.002$ and $V = 0.002 \text{ m}^3$

For the given P : $v_f < v < v_g$ so 2-phase T = Tsat = 158.85 °C

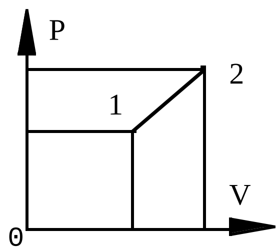
Work is done while piston moves at $P_{lift}$ = constant = 300 kPa so we get

$${}_1W_2 = \int P\, dV = m\, P_{lift}(v_2 - v_1) = 1 \times 300(0.002 - 0.001002) = \mathbf{0.30\ kJ}$$

**4.13** Air in a spring loaded piston/cylinder has a pressure that is linear with volume, $P = A + BV$. With an initial state of P = 150 kPa, V = 1 L and a final state of 800 kPa and volume 1.5 L it is similar to the setup in Problem 3.16. Find the work done by the air.

Solution:

Knowing the process equation: P = A + BV giving a linear variation of pressure versus volume the straight line in the P-V diagram is fixed by the two points as state 1 and state 2. The work as the integral of PdV equals the area under the process curve in the P-V diagram.




State 1: $P_1 = 150$ kPa $\quad V_1 = 1 \text{ L} = 0.001 \text{ m}^3$

State 2: $P_2 = 800$ kPa $\quad V_2 = 1.5 \text{ L} = 0.0015 \text{ m}^3$

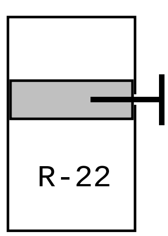
Process: P = A + BV linear in V

$$\Rightarrow {}_1W_2 = \int_1^2 PdV = \left(\frac{P_1 + P_2}{2}\right)(V_2 - V_1)$$

$$= \frac{1}{2}(150 + 800)(1.5 - 1) \times 0.001 = \mathbf{0.2375\ kJ}$$

**4.31** A vertical cylinder (Fig. P4.31) has a 90-kg piston locked with a pin trapping 10 L of R-22 at 10°C, 90% quality inside. Atmospheric pressure is 100 kPa, and the cylinder cross-sectional area is 0.006 $m^2$. The pin is removed, allowing the piston to move and come to rest with a final temperature of 10°C for the R-22. Find the final pressure, final volume and the work done by the R-22.

Solution:




State 1: (T,x) from table B.4.1

$v_1 = 0.0008 + 0.9 \times 0.03391 = 0.03132$

$m = V_1/v_1 = 0.010/0.03132 = 0.319$ kg

Force balance on piston gives the equilibrium pressure

$$P_2 = P_0 + m_P g/ A_P = 100 + \frac{90 \times 9.807}{0.006 \times 1000} = \mathbf{247\ kPa}$$

State 2: (T,P) interpolate $V_2 = mv_2 = 0.319 \times 0.10565 = 0.0337 \text{ m}^3 = \mathbf{33.7\ L}$

$$W_{12} = \int P_{equil}\, dV = P_2(V_2 - V_1) = 247(0.0337 - 0.010) = \mathbf{5.85\ kJ}$$

**4.45** Consider the process of inflating a helium balloon, as described in Problem 3.14. For a control volume that consists of the space inside the balloon, determine the work done during the overall process.

Solution :

Inflation at constant $P = P_0 = 100$ kPa to $D_1 = 1$ m, then

$$P = P_0 + C\,(D^{*\,-1} - D^{*\,-2}), \qquad D^* = D / D_1,$$

to $D_2 = 4$ m, $P_2 = 400$ kPa, from which we find the constant C as:

$$400 = 100 + C[\,(1/4) - (1/4)^2\,] \quad => \quad C = 1600$$

The volumes are: $V = \frac{\pi}{6} D^3 \quad => \quad V_1 = 0.5236\ m^3; \quad V_2 = 33.51\ m^3$

$$W_{CV} = P_0(V_1 - 0) + \int_1^2 PdV$$

$$= P_0(V_1 - 0) + P_0(V_2 - V_1) + \int_1^2 C(D^{*\,-1} - D^{*\,-2})dV$$

$$V = \frac{\pi}{6} D^3, \qquad dV = \frac{\pi}{2} D^2\, dD \;=\; \frac{\pi}{2} D_1^3\, D^{*\,2}\, dD^*$$

$$\Rightarrow W_{CV} = P_0 V_2 + 3CV_1 \int_{D_1^*=1}^{D_2^*=4} (D^*-1)dD^*$$

$$= P_0 V_2 + 3CV_1\Big[\frac{D_2^{*\,2} - D_1^{*\,2}}{2} - (D_2^* - D_1^*)\Big]_1^4$$

$$= 100 \times 33.51 + 3 \times 1600 \times 0.5236\Big[\frac{16-1}{2} - (4-1)\Big]$$

$= \mathbf{14661\ kJ}$