ME 24-221
THERMODYNAMICS I

Solutions to extra problems in Chapter 11:
November 29, 2000
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11.3 A utility runs a Rankine cycle with a water boiler at 3.5 MPa and the cycle has the highest and lowest temperatures of 450°C and 45°C respectively. Find the plant efficiency and the efficiency of a Carnot cycle with the same temperatures.

Solution:

1: 45^oC , $x = 0$ => $h_1 = 188.42$, $v_1 = 0.00101$, $P_{sat} = 9.6$ kPa

3: 3.5 MPa , 450^oC => $h_3 = 3337.2$, $s_3 = 7.0051$

C.V. Pump Rev adiabatic

$-w_P = h_2 - h_1$; $s_2 = s_1$

since incompressible it is easier to find work as

$-w_p = \int v\, dP = v_1 (P_2 - P_1) = 0.00101 (3500 - 9.6) = 3.525$

=> $h_2 = h_1 - w_p = 188.42 + 3.525 = 191.95$

C.V. Boiler : $q_h = h_3 - h_2 = 3337.2 - 191.95 = 3145.3$

C.V. Turbine : $w_t = h_3 - h_4$; $s_4 = s_3$

$s_4 = s_3 = 7.0051 = 0.6386 + x_4 (7.5261)$ => $x_4 = 0.8459$

=> $h_4 = 188.42 + 0.8459 (2394.77) = 2214.2$

$w_t = 3337.2 - 2214.2 = 1123$ kJ/kg

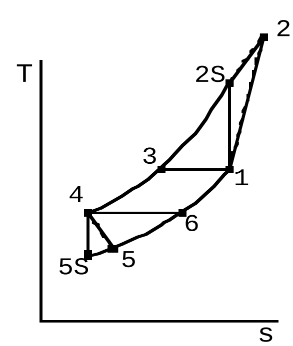
C.V. Condenser : $q_L = h_4 - h_1 = 2214.2 - 188.42 = 2025.78$ kJ/kg

$\eta_{cycle} = w_{net} / q_H = (w_t + w_p) / q_H = (1123 - 3.5) / 3145.3 =$ **0.356**

$\eta_{carnot} = 1 - T_L / T_H = 1 - \dfrac{273.15 + 45}{273.15 + 450} =$ **0.56**

11.102 Repeat Problem 11.101, but assume an isentropic efficiency of 75% for both the compressor and the expander.

From solution 11.101 :



$T_{2S} = 613$ K, $w_{SC} = 326$ kJ/kg

$T_{5S} = 104.9$ K, $w_{SE} = 118.7$

$\Rightarrow$ $w_C = w_{SC} / \eta_{SC} = 326/0.75 = 434.6$ kJ/kg

$w_E = \eta_{SE} \times w_{SE} = 0.75 \times 118.7 = 89.0$ kJ/kg

$= C_{P0}(T_4 - T_5) = 1.004(223.2 - T_5)$

$\Rightarrow$ $T_5 = 134.5$ K

$w_{NET} = 89.0 - 434.6 = -345.6$ kJ/kg

$q_L = C_{P0}(T_6 - T_5) = 1.004(223.2 - 134.5) = 89.0$ kJ/kg

$\beta = q_L/(-w_{NET}) = 89.0/345.6 =$ **0.258**

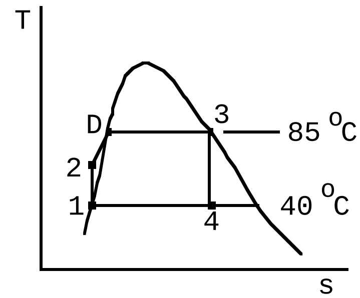
11.5 A supply of geothermal hot water is to be used as the energy source in an ideal Rankine cycle, with R-134a as the cycle working fluid. Saturated vapor R-134a leaves the boiler at a temperature of 85°C, and the condenser temperature is 40°C. Calculate the thermal efficiency of this cycle.

CV: Pump (use R-134a Table B.5)

$$-w_P = h_2 - h_1 = \int_1^2 v dP \approx v_1(P_2 - P_1)$$

$$= 0.000873(2926.2 - 1017.0) = 1.67 \text{ kJ/kg}$$

$$h_2 = h_1 - w_P = 256.54 + 1.67 = 258.21 \text{ kJ/kg}$$



CV: Boiler

$$q_H = h_3 - h_2 = 428.10 - 258.21 = 169.89 \text{ kJ/kg}$$

CV: Turbine

$$s_4 = s_3 = 1.6782 = 1.1909 + x_4 \times 0.5214 \quad \Rightarrow \quad x_4 = 0.9346$$

$$h_4 = 256.54 + 0.9346 \times 163.28 = 409.14$$

Energy Eq.: $w_T = h_3 - h_4 = 428.1 - 409.14 = 18.96$ kJ/kg

$$w_{NET} = w_T + w_P = 18.96 - 1.67 = 17.29 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 17.29/169.89 = \mathbf{0.102}$$

11.14 Consider an ideal Rankine cycle using water with a high-pressure side of the cycle at a supercritical pressure. Such a cycle has a potential advantage of minimizing local temperature differences between the fluids in the steam generator, such as the instance in which the high-temperature energy source is the hot exhaust gas from a gas-turbine engine. Calculate the thermal efficiency of the cycle if the state entering the turbine is 25 MPa, 500°C, and the condenser pressure is 5 kPa. What is the steam quality at the turbine exit?

$s_4 = s_3 = 5.9592 = 0.4764 + x_4 \times 7.9187$

$x_4 = \mathbf{0.6924}$

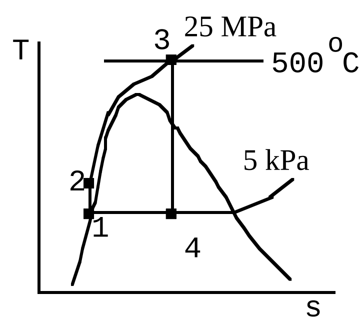
Very low for a turbine exhaust

$s_1 = 0.4764$, $h_1 = 137.82$

$h_4 = 1816$, $h_3 = 3162.4$

$s_2 = s_1 \Rightarrow h_2 = 162.8$

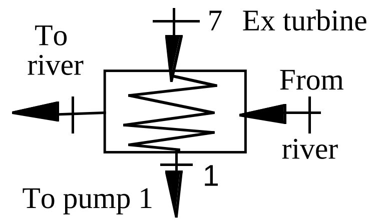
$w_{NET} = h_3 - h_4 - (h_2 - h_1) = 1321.4$



$q_H = h_3 - h_2 = 2999.6$, $\eta = w_{NET}/q_H = \mathbf{0.44}$

11.23 A steam power plant operates with a boiler output of 20 kg/s steam at 2 MPa, 600°C. The condenser operates at 50°C dumping energy to a river that has an average temperature of 20°C. There is one open feedwater heater with extraction from the turbine at 600 kPa and its exit is saturated liquid. Find the mass flow rate of the extraction flow. If the river water should not be heated more than 5°C how much water should be pumped from the river to the heat exchanger (condenser)?

Solution: The setup is as shown in Fig. 11.10, Condenser:



1: 50^oC sat liq. $v_1 = 0.001012$, $h_1 = 209.31$

2: 600 kPa $s_2 = s_1$

3: 600 kPa sat liq. $h_3 = h_f = 670.54$

5: P , T $h_5 = 3690.1$ $s_5 = 7.7023$

6: 600 kPa $s_6 = s_5$ => $h_6 = 3270.0$

CV P1

$$w_{P1} = -v_1(P_2 - P_1) = -0.001012\ (600 - 12.35) = -0.595$$

$$h_2 = h_1 - w_{P1} = 209.9$$

C.V FWH

$$x\ h_6 + (1 - x)\ h_2 = h_3$$

$$x = \frac{h_3 - h_2}{h_6 - h_2} = \frac{670.54 - 209.9}{3270.0 - 209.9} = \mathbf{0.1505}$$

CV Turbine

$$s_7 = s_6 = s_5 \quad => \quad x_7 = 0.9493\ , \qquad h_7 = 2471.17 \text{ kJ/kg}$$

$$q_L = h_7 - h_1 = 2471.17 - 209.31 = 2261.86 \text{ kJ/kg}$$

$$\dot{Q}_L = (1 - x)\ \dot{m} q_L = 0.85 \times 20 \times 2261.86 = 38429 \text{ kW}$$

$$= \dot{m}_{H2O}\ \Delta h_{H2O} = \dot{m}\ (20.93) = 38429 \text{ kW}$$

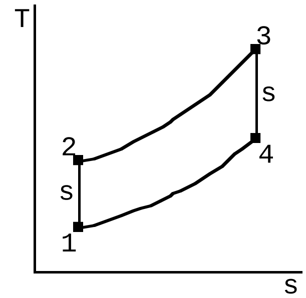
$$\dot{m} = \mathbf{1836\ kg/s}$$

CV Condenser + Heat Exchanger

$$0 = \dot{m}_{H2O}\ (s_7 - s_1)\ (1 - x) - \frac{\dot{Q}_L}{T_L} + \dot{S}_g$$

$$\dot{S}_g = \frac{38429}{293.15} - 20 \times 0.85 \times (7.7023 - 0.7037) = \mathbf{12.184\ kW/K}$$

11.50 A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the maximum temperature is 1600 K. The minimum pressure in the cycle is 100 kPa, and the compressor pressure ratio is 14 to 1. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?



$T_1 = 300$ K, $P_2/P_1 = 14$, $T_3 = 1600$ K

a) Assume const C_{P0}: $s_2 = s_1$

$$\Rightarrow T_2 = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300(14)^{0.286} = 638.1 \text{ K}$$

$$-w_C = -w_{12} = h_2 - h_1 = C_{P0}(T_2 - T_1)$$

$$= 1.004\ (638.1 - 300) = 339.5 \text{ kJ/kg}$$

Also, $s_4 = s_3 \rightarrow T_4 = T_3(P_4/P_3)^{\frac{k-1}{k}} = 1600\ (1/14)^{0.286} = 752.2$ K

$w_T = w_{34} = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004\ (1600 - 752.2) = 851.2$ kJ/kg

$w_{NET} = 851.2 - 339.5 = 511.7$ kJ/kg

$\dot{m} = \dot{W}_{NET}/w_{NET} = 100000/511.7 = 195.4$ kg/s

$\dot{W}_T = \dot{m}w_T = 195.4 \times 851.2 =$ **166.32 MW**

$-w_C/w_T = 339.5/851.2 =$ **0.399**

b) $q_H = C_{P0}(T_3 - T_2) = 1.004\ (1600 - 638.1) = 965.7$ kJ/kg

$\eta_{TH} = w_{NET}/q_H = 511.7/965.7 =$ **0.530**

11.78 A diesel engine has a compression ratio of 20:1 with an inlet of 95 kPa, 290 K, state 1, with volume 0.5 L. The maximum cycle temperature is 1800 K. Find the maximum pressure, the net specific work and the thermal efficiency.

$$P_2 = 95 \times (20)^{1.4} = 6297.5 \text{ kPa}$$

$$T_2 = T_1(v_1 / v_2)^{k-1} = 290 \times 20^{0.4} = 961 \text{ K}$$

$$P_3 = P_2(T_3 / T_2) = 6297.5 \times (1800 / 961) = \mathbf{11796\ kPa}$$

$$-{}_1w_2 = u_2 - u_1 \approx C_{vo}(T_2 - T_1) = 0.717(961 - 290) = 481.1 \text{ kJ / kg}$$

$$T_4 = T_3(v_3 / v_4)^{0.4} = 1800(1 / 20)^{0.4} = 543 \text{ K}$$

$${}_3w_4 = u_3 - u_4 \approx C_{vo}(T_3 - T_2) = 0.717(1800 - 543) = 901.3 \text{ kJ / kg}$$

$$w_{net} = {}_3w_4 + {}_1w_2 = 901.3 - 481.1 = \mathbf{420.2\ kJ / kg}$$

$$\eta = w_{net} / q_H = 1 - (1 / 20)^{0.4} = \mathbf{0.698} \qquad (= 420.2 / 0.717(1800 - 961))$$

11.54 The gas-turbine cycle shown in Fig. P11.54 is used as an automotive engine. In the first turbine, the gas expands to pressure P_5, just low enough for this turbine to drive the compressor. The gas is then expanded through the second turbine connected to the drive wheels. The data for the engine are shown in the figure and assume that all processes are ideal. Determine the intermediate pressure P_5, the net specific work output of the engine, and the mass flow rate through the engine. Find also the air temperature entering the burner T_3, and the thermal efficiency of the engine.

a) $s_2 = s_1 \Rightarrow T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300(6)^{0.286} = 500.8 \text{ K}$

$-w_C = -w_{12} = C_{P0}(T_2 - T_1) = 1.004(500.8 - 300) = 201.6 \text{ kJ/kg}$

$w_{T1} = -w_C = 201.6 = C_{P0}(T_4 - T_5) = 1.004(1600 - T_5)$

$\Rightarrow T_5 = 1399.2 \text{ K}$

$s_5 = s_4 \Rightarrow P_5 = P_4\left(\frac{T_5}{T_4}\right)^{\frac{k-1}{k}} = 600\left(\frac{1399.2}{1600}\right)^{3.5} = \mathbf{375\ kPa}$

b) $s_6 = s_5 \Rightarrow T_6 = T_5\left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} = 1399.2\left(\frac{100}{375}\right)^{0.286} = 958.8 \text{ K}$

$w_{T2} = C_{P0}(T_5 - T_6) = 1.004(1399.2 - 958.8) = 442.2 \text{ kJ/kg}$

$\dot{m} = \dot{W}_{NET}/w_{T2} = 150/442.2 = \mathbf{0.339\ kg/s}$

c) Ideal regenerator $\Rightarrow$ $T_3 = T_6 = \mathbf{958.8\ K}$

$q_H = C_{P0}(T_4 - T_3) = 1.004(1600 - 958.8) = 643.8 \text{ kJ/kg}$

$\eta_{TH} = w_{NET}/q_H = 442.2/643.8 = \mathbf{0.687}$

11.60 A two-stage air compressor has an intercooler between the two stages as shown in Fig. P11.60. The inlet state is 100 kPa, 290 K, and the final exit pressure is 1.6 MPa. Assume that the constant pressure intercooler cools the air to the inlet temperature, $T_3 = T_1$. It can be shown, see Problem 9.130, that the optimal pressure, $P_2 = (P_1P_4)^{1/2}$, for minimum total compressor work. Find the specific compressor works and the intercooler heat transfer for the optimal P_2.

Optimal intercooler pressure $P_2 = \sqrt{100 \times 1600} = 400$ kPa

1: $h_1 = 290.43$, $P_{r1} = 0.9899$

C.V.: C1 $-w_{C1} = h_2 - h_1$, $s_2 = s_1$

$\Rightarrow$ $P_{r2} = P_{r1}(P_2/P_1) = 3.9596$, $T_2 = 430$, $h_2 = 431.95$

$-w_{C1} = 431.95 - 290.43 =$ **141.5 kJ/kg**

C.V.Cooler: $T_3 = T_1$ $\Rightarrow$ $h_3 = h_1$

$q_{OUT} = h_2 - h_3 = h_2 - h_1 = -w_{C1} =$ **141.5 kJ/kg**

C.V.: C2 $T_3 = T_1$, $s_4 = s_3$

$\Rightarrow$ $P_{r4} = P_{r3}(P_4/P_3) = P_{r1}(P_2/P_1) = 3.9596$, $T_4 = T_2$

Thus we get $-w_{C2} = -w_{C1} =$ **141.5 kJ/kg**

11.72 Repeat Problem 11.71, but assume variable specific heat. The ideal gas air tables, Table A.7, are recommended for this calculation (and the specific heat from Fig. 5.10 at high temperature).

Table A.7 is used with interpolation.

a) $T_1 = 283.15$ K, $u_1 = 202.3$, $v_{r1} = 207.94$

$s_2 = s_1$ $\Rightarrow$ $v_{r2} = v_{r1}(v_2/v_1) = 207.94(1/7) = 29.705$

$\Rightarrow$ $T_2 = 606.7$ K, $u_2 = 440.2$ => $-w_{12} = u_2 - u_1 = 237.9$,

$u_3 = 440.2 + 1800 = 2240.2$ => $T_3 =$ **2575.5 K**, $v_{r3} = 0.3402$

$P_3 = 90 \times 7 \times 2575.5 / 283.15 =$ **5730 kPa**

b) $v_{r4} = v_{r3} \times 7 = 2.3814$ => $T_4 = 1437$ K; $u_4 = 1147$

${}_3w_4 = u_3 - u_4 = 2240.2 - 1147 = 1093.2$

Ë $w_{net} = 1093.2 - 237.9 = 855.3$ kJ/kg

$\eta_{TH} = w_{net} / q_H = 855.3 / 1800 =$ **0.475**

(c) mep = 855.3 / (0.9029 - 0.129) = **1105 kPa**

11.81 Consider an ideal air-standard diesel cycle in which the state before the compression process is 95 kPa, 290 K, and the compression ratio is 20. Find the maximum temperature (by iteration) in the cycle to have a thermal efficiency of 60%?

Diesel cycle: $P_1 = 95$ kPa, $T_1 = 290$ K, $v_1/v_2 = 20$, $\eta_{TH} = 0.6$

$$T_2 = T_1(v_1/v_2)^{k-1} = 290(20)^{0.4} = 961.2 \text{ K}$$

$$v_1 = 0.287 \times 290/95 = 0.876 = v_4, \qquad v_2 = 0.876 / 20 = 0.0438$$

$$v_3 = v_2(T_3/T_2) = 0.043883(T_3/961.2) = 0.0000456 T_3$$

$$T_3 = T_4\,(v_4/v_3)^{k-1} = \left(\frac{0.876}{0.0000456\,T_3}\right)^{0.4} \;\Rightarrow\; T_4 = 0.019345\,T_3^{1.4}$$

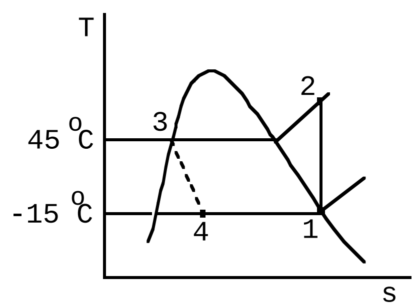
$$\eta_{TH} = 0.60 = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{0.019345 \times T_3^{1.4} - 290}{1.4(T_3 - 961.2)}$$

$$\Rightarrow\; 0.019345 \times T_3^{1.4} - 0.56 \times T_3 + 248.272 = 0$$

3050: LHS = +1.06 3040: LHS = -0.036,

$T_3 =$ **3040 K**

11.88 The environmentally safe refrigerant R-134a is one of the replacements for R-12 in refrigeration systems. Repeat Problem 11.87 using R-134a and compare the result with that for R-12.



$h_1 = 389.2$, $s_2 = s_1 = 1.7354$

$h_3 = 264.11$, $P_3 = P_2 = 1.16$ MPa

At 1 MPa, $T_2 = 45.9$, $h_2 = 426.8$

At 1.2 MPa, $T_2 = 53.3$, $h_2 = 430.7$

$\Rightarrow$ $T_2 = 51.8$, $h_2 = 429.9$

$-w_C = h_2 - h_1 = 429.9 - 389.2$

$= 40.7$ kJ/kg

$q_L = h_1 - h_4 = h_1 - h_3 = 389.2 - 264.11 = 125.1$ kJ/kg

$\beta = q_L/(-w_C) = 125.1/40.7 =$ **3.07**

11.94 The air conditioner in a car uses R-134a and the compressor power input is 1.5 kW bringing the R-134a from 201.7 kPa to 1200 kPa by compression. The cold space is a heat exchanger that cools atmospheric air from the outside down to 10°C and blows it into the car. What is the mass flow rate of the R-134a and what is the low temperature heat transfer rate. How much is the mass flow rate of air at 10°C?

Standard Refrigeration Cycle

Table B.5 $h_1 = 392.285$; $s_1 = 1.7319$; $h_4 = h_3 = 266$

C.V. Compressor (assume ideal)

$\dot{m}_1 = \dot{m}_2$ $\quad w_C = h_2 - h_1$; $s_2 = s_1 + s_{gen}$

P_2, $s = s_1$ -> $h_2 = 429.5$ -> $w_C = 37.2$

$\dot{m}\, w_C = \dot{W}_C$ -> $\dot{m} = 1.5 / 37.2 = 0.0403$ kg /s

C.V. Evaporator

$\dot{Q}_L = \dot{m}(h_1 - h_4) = 0.0405(392.28 - 266) = 5.21$ kW

C.V. Air Cooler

$\dot{m}_{air}\Delta h_{air} = \dot{Q}_L \approx \dot{m}_{air}C_p\Delta T$

$\dot{m}_{air} = \dot{Q}_L / (C_p\Delta T) = 5.21 / (1.004\times20) =$ **0.26 kg / s**

11.97 The refrigerant R-22 is used as the working fluid in a conventional heat pump cycle. Saturated vapor enters the compressor of this unit at 10°C; its exit temperature from the compressor is measured and found to be 85°C. If the isentropic efficiency of the compressor is estimated to be 70%, what is the coefficient of performance of the heat pump?

R-22 heat pump:

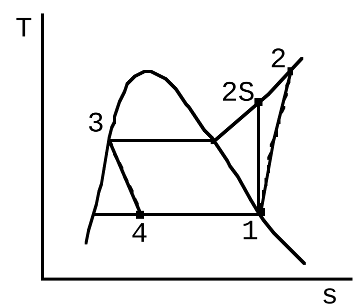
$T_{EVAP} = 10\,^oC$, $\eta_{S\,COMP} = 0.70$

$T_2 = 85\,^oC$

Isentropic compressor:

$s_{2S} = s_1 = 0.9129$

but P_2 unknown. Trial & error.



Assume $P_2 = 2.11$ MPa

At P_2, s_{2S}: $T_{2S} = 72.1^oC$, $h_{2S} = 281.59$, At P_2, T_2: $h_2 = 293.78$

calculate $\eta_{S\,COMP} = (h_{2S} - h_1)/(h_2 - h_1) = \dfrac{281.59 - 253.42}{293.28 - 253.42} = 0.698 \approx 0.70$

OK $\Rightarrow$ $P_2 = 2.11\text{ MPa} = P_3$

$\Rightarrow$ $T_3 = 53.7\,^oC$, $h_3 = 112.99$, $w_C = h_2 - h_1 = 40.36$

$q_H = h_2 - h_3 = 180.79$, $\beta' = q_H/w_C =$ **4.48**