

ME 24-221 Thermodynamics I

Solution to Assignment No: 1
Due Date: 8 September 2000
Fall 2000
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- 2.12** A 5 m³ container is filled with 900 kg of granite (density 2400 kg/m³) and the rest of the volume is air with density 1.15 kg/m³. Find the mass of air and the overall (average) specific volume.

Solution:

$$\begin{aligned} m_{\text{air}} &= \rho V = \rho_{\text{air}} (V_{\text{tot}} - m_{\text{granite}} / \rho) \\ &= 1.15 [5 - (900 / 2400)] = 1.15 \times 4.625 = \mathbf{5.32 \text{ kg}} \\ v &= V / m = 5 / (900 + 5.32) = \mathbf{0.00552 \text{ m}^3/\text{kg}} \end{aligned}$$

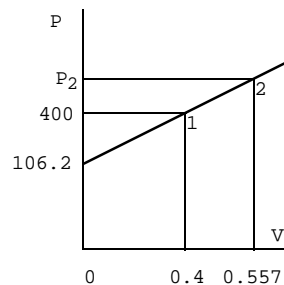
- 2.22** A 5-kg piston in a cylinder with diameter of 100 mm is loaded with a linear spring and the outside atmospheric pressure of 100 kPa. The spring exerts no force on the piston when it is at the bottom of the cylinder and for the state shown, the pressure is 400 kPa with volume 0.4 L. The valve is opened to let some air in, causing the piston to rise 2 cm. Find the new pressure.

Solution:

A linear spring has a force linear proportional to displacement. $F = kx$, so the equilibrium pressure then varies linearly with volume: $P = a + bV$, with an intercept a and a slope $b = dP/dV$. Look at the balancing pressure at zero volume ($V \rightarrow 0$) when there is no spring force $F = PA = P_oA + m_p g$ and the initial state.

These two points determine the straight line shown in the P-V diagram.

$$\text{Piston area} = A_p = (\pi/4) \times 0.1^2 = 0.00785 \text{ m}^2$$



$$\begin{aligned} a &= P_0 + \frac{m_p g}{A_p} = 100 + \frac{5 \times 9.80665}{0.00785} \\ &= 106.2 \text{ kPa} \quad \text{intersect for zero volume.} \end{aligned}$$

$$V_2 = 0.4 + 0.00785 \times 20 = 0.557 \text{ L}$$

$$\begin{aligned} P_2 &= P_1 + \frac{dP}{dV} \Delta V \\ &= 400 + \frac{(400-106.2)}{0.4-0} (0.557 - 0.4) \\ &= \mathbf{515.3 \text{ kPa}} \end{aligned}$$

- 2.33** Two hydraulic piston/cylinders are of same size and setup as in Problem 2.32, but with negligible piston masses. A single point force of 250 N presses down on piston A. Find the needed extra force on piston B so that none of the pistons have to move.

Solution:

No motion in connecting pipe: $P_A = P_B$ & Forces on pistons balance

$$A_A = 75 \text{ cm}^2; \quad A_B = 25 \text{ cm}^2$$

$$P_A = P_0 + F_A / A_A = P_B = P_0 + F_B / A_B$$

$$F_B = F_A A_B / A_A = 250 \times 25 / 75 = \mathbf{83.33 \text{ N}}$$

- 2.36** Two cylinders are connected by a piston as shown in Fig. P2.36. Cylinder A is used as a hydraulic lift and pumped up to 500 kPa. The piston mass is 25 kg and there is standard gravity. What is the gas pressure in cylinder B?

Solution:

$$\text{Force balance for the piston: } P_B A_B + m_p g + P_0 (A_A - A_B) = P_A A_A$$

$$A_A = (\pi/4)0.1^2 = 0.00785 \text{ m}^2; \quad A_B = (\pi/4)0.025^2 = 0.000491 \text{ m}^2$$

$$P_B A_B = P_A A_A - m_p g - P_0 (A_A - A_B) = 500 \times 0.00785 - (25 \times 9.807/1000) - 100 (0.00785 - 0.000491) = 2.944 \text{ kN}$$

$$P_B = 2.944/0.000491 = 5996 \text{ kPa} = \mathbf{6.0 \text{ MPa}}$$

- 2.37** Two cylinders are filled with liquid water, $\rho = 1000 \text{ kg/m}^3$, and connected by a line with a closed valve. A has 100 kg and B has 500 kg of water, their cross-sectional areas are $A_A = 0.1 \text{ m}^2$ and $A_B = 0.25 \text{ m}^2$ and the height h is 1 m. Find the pressure on each side of the valve. The valve is opened and water flows to an equilibrium. Find the final pressure at the valve location.

Solution:

$$V_A = v_{\text{H}_2\text{O}} m_A = m_A / \rho = 0.1 = A_A h_A \quad \Rightarrow \quad h_A = 1 \text{ m}$$

$$V_B = v_{\text{H}_2\text{O}} m_B = m_B / \rho = 0.5 = A_B h_B \quad \Rightarrow \quad h_B = 2 \text{ m}$$

$$P_{VB} = P_0 + \rho g (h_B + H) = 101325 + 1000 \times 9.81 \times 3 = 130755 \text{ Pa}$$

$$P_{VA} = P_0 + \rho g h_A = 101325 + 1000 \times 9.81 \times 1 = 111135 \text{ Pa}$$

Equilibrium: same height over valve in both

$$V_{\text{tot}} = V_A + V_B = h_2 A_A + (h_2 - H) A_B \Rightarrow h_2 = \frac{h_A A_A + (h_B + H) A_B}{A_A + A_B} = 2.43 \text{ m}$$

$$P_{V2} = P_0 + \rho g h_2 = 101.325 + (1000 \times 9.81 \times 2.43)/1000 = \mathbf{125.2 \text{ kPa}}$$