Department of Mathematical Sciences
Carnegie Mellon University

Spring 2002

21-256 Multivariate Analysis and Approximation

Assignment 6

Solutions to *all* the following problems (except the Practice Problems) should be written up and handed in to your TA.

Due in recitation on Thursday, February 28, 2002

Walker:

Section 2.7: Problems 2, 4

Stewart:

Section 14.1: Problems 6, 12, 30 (do not do part f), 38

Section 14.2: Problems 5, 8

Some Practice Problems: You do not need to hand in the solutions to the following problems.

1. Consider the following system of linear equations:

$$\begin{aligned} x_2 + 2x_3 + 2x_4 &= 5 \\ x_1 + 3x_2 + 5x_3 + 5x_4 &= 14 \\ x_1 + x_2 + x_3 + x_4 &= 4 \\ 2x_1 + 3x_2 + 4x_3 + 4x_4 &= 15 \end{aligned}$$

- **a.** Find a particular solution to this system.
- **b.** What is the coefficient matrix for this system?
- **c.** What is the associated homogeneous system?
- **d.** Use Gaussian elimination to find all solutions to the associated homogeneous system.
- **e.** Find a set of vectors whose span is the null space of the coefficient matrix.
- **f.** Find all solutions to the system of equations.
- **g.** What is the nullity and rank of the coefficient matrix?

2. Repeat parts **b.** through **f.** of problem **1.** for the following system of linear equations:

$$\begin{aligned} 2x_1 - 4x_3 &= 8 \\ x_1 - 2x_2 - 2x_3 &= 14 \\ x_1 + x_2 - 2x_3 &= -1 \\ 3x_1 + x_2 + x_3 &= 0 \end{aligned}$$

3. Consider the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 5 & -2 & -9 \\ 3 & 1 & -1 \\ 3 & -2 & -7 \end{bmatrix}$$

a. Find a set of vectors whose span is the null space of A.

b. What is the nullity and rank of A.

4. The Splat manufacturing company produces three products, Whamies, Zoomies, and Clompers. The profits for each unit of Whamies, Zoomies, and Clompers sold are \$1, \$2, and \$3, respectively. The fixed costs are \$17,000 per year and the costs of producing each unit of Whamies, Zoomies, and Clompers are \$4, \$5, and \$7, respectively. Next year, the total of all three products to be produced and sold is 11,000 units, and a total profit of \$25,000 is to be realized. If the total cost is to be \$80,000, how many units of each of the products should be produced next year?

5. Find the inverses for each of the following matrices

a. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 7 & 0 & -2 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$ **c.** $\begin{bmatrix} 2 & 1 & 0 \\ 4 & -1 & 5 \\ 1 & -1 & 2 \end{bmatrix}$

6. Which of the matrices in problem **5.** is positive definite? (the definition for positive definiteness can be found in Walker's text: Section 2.5, problem 12.)

7. For each of the following systems, if the coefficient matrix is invertible, solve the system by using the inverse. If not, solve the system by using Gaussian elimination.

a.

$$\begin{aligned} x_1 + 3x_2 + 3x_3 &= 7 \\ 2x_1 + x_2 + x_3 &= 4 \\ x_1 + x_2 + x_3 &= 4 \end{aligned}$$

b.

$$\begin{aligned} x_1 + 2x_3 + x_4 &= 4 \\ x_1 - x_2 + 2x_4 &= 12 \\ 2x_1 + x_2 + x_4 &= 12 \\ x_1 + 2x_2 + x_3 + x_4 &= 12 \end{aligned}$$

8. Evaluate each of the following:

a. $\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$ **b.** $\begin{vmatrix} 2 & 1 & 3 \\ 2 & 0 & 1 \\ -4 & 0 & 6 \end{vmatrix}$ **c.** $\begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & -1 \\ 1 & 2 & -3 \end{vmatrix}$