21-228 Homework 2

Due September 18, 2001

1 Suggested Exercises (not for submission!)

These are more "exercise" level problems that you should practice as much as time allows.

In particular look at 1.3.19.

- 1.3.1 through 1.3.10 of Bogart, except #2. 1.3.19 as well
- 1.4.1, 1.4.3, 1.4.9, 1.4.10, 1.4.16

2 Problems to be Submitted

Expect this to take more time than HW1. As usual, justify all answers.

1. Without appealing to the formula for $\binom{n}{k}$, prove $\binom{n}{m}\binom{n-m}{k} = \binom{n}{k}\binom{n-k}{m}$.

2. Prove the formula:

$$\sum_{j=0}^{k} \binom{m}{j}\binom{n}{k-j} = \binom{m+n}{k},$$

again without using the formula for a binomial coefficient.

3. What value of k makes $\binom{n}{k}$ maximized for a given value of n? Prove your answer. To do this, consider the relationship between $\binom{n}{k}$ and $\binom{n}{k+1}$ and figure out in which cases the former is larger than the latter.

4. Show that:

$$\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}$$

5. A person wants to walk from a certain point in a city to a point seven blocks north and eight blocks east. In how many ways may she go from one point to the other and walk exactly 17 blocks? In how many ways may she go from one point to the other and walk exactly 16 blocks?

6. Suppose we have a standard deck of playing cards, and we use only the clubs. How many ways can we, to four people, distribute the 13 cards so that each person gets at least 3 of them? What is the total number of ways to distribute the 13 cards? This gives us a rough (but not quite right) idea of how likely it is that if we deal out all 52 cards to the four people, with each person getting 13 of them, that each one will get at least three clubs.

7. Show that:

$$\sum_{k=0}^{n} \frac{2n!}{k!^2(n-k)!^2} = \binom{2n}{n}^2$$