

Concepts of Math: Recitation 8

September 22, 2015

More Induction

1. Prove that $\forall n, k > 0, F_{n+k} = F_{n+1}F_k + F_nF_{k-1}$, where F_n are the Fibonacci numbers defined by $F_1 = 1, F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n > 2$.

2. Prove that $\forall n \in \mathbb{N}, n \geq 2$

$$1 + \frac{1}{2^3} + \frac{1}{3^3} + \cdots + \frac{1}{n^3} < \frac{3}{2} - \frac{1}{n^2}.$$

3. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence satisfying $a_1 = a_2 = 1$ and

$$a_n = \frac{1}{2} \left(a_{n-1} + \frac{2}{a_{n-2}} \right) \quad \text{for } n \geq 3.$$

Prove that $1 \leq a_n \leq 2$ for $n \in \mathbb{N}$.

4. Consider n married couples at a party. Suppose that no person shakes hands with his or her spouse, and the $2n - 1$ people other than the host shake hands with different numbers of people. With how many people does the hostess shake hands? First conjecture your answer, then prove it by induction.